Code Time Division Multiple Access for Multicarrier Communication Systems

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Abstract

Recently, much attention has been drawn to high-rate high-quality wireless mobile communications. Multicarrier modulation is proposed to deliver such a stringent service requirement. On the down link of a multicarrier system, orthogonal codes are assigned to users for data transmission. We can view these codes as code channels. In a wireless communication system, since different users experience different fading characteristics, different time division multiple access to the code channels will give different performance. Therefore, an important problem is to make multiple users share these channels in time domain optimally. Optimization of the system can be considered under different criteria. For a system where the applications are delay sensitive, we optimize it by minimizing the total transmission power while the data rate requirement for each user is guaranteed. Simulations show that major performance improvement can be achieved via our near optimal time division approach, comparing with arbitrary time division. For simplification, we propose a class of ad hoc approaches, which assume one user will exclusively occupy a code channel in the whole time slot once the channel is assigned to it. Simulation results show that the ad hoc approach yields performance close to the near optimal approach. We also derive a lower bound on the optimal solution and show that, when the system capacity becomes large, the performance of the near optimal and the ad hoc approaches would converge to the lower bound.

We also consider the implementation of an ad hoc approach called one-to-one approach, which assign one and only one code to each user in a whole transmission time slot. We also propose a realistic wireless mobile fading channel model before we consider the system optimization. We optimize the system by minimize the total transmission power while the target SNR of each user is guaranteed. Results show that the performance depends heavily on the set of available codes. To investigate the effects of the code sets, we consider the Hadamard codes as well as random orthogonal codes. Simulations show that random code sets with high order of selection diversity enjoy much better performance with optimal code assignment. To investigate the practicality of the approach, we also consider the effects of quantization and the rate of optimization. Results show that the approach is practical and effective. Major improvements can be obtained with little quantized feedback information and lower optimization rate.

摘要

當前，高數據率高質量的無綫移動通訊業務受到人們越來越多的關註。為此，人們提出了多載波調製方案來滿足這種迫切的需求。在多載波通訊系統的下行通道中，正交碼被分配給用戶來接受資料。這些碼可以被看成碼通道。在無線通訊系統中，因爲不同的用戶所經歷的衰減不一樣，不同的時分複用碼通道的方法有不同的系統性能。因此，在多載波通訊系統中，一個很重要的問題就是如何讓多用戶在時間域中最優地共用多個碼通道。 我們可以基於不同的標準來考慮這個優化問題。在一個用戶對延遲很敏感的系統中，在保證每個用戶的資料率得到保證的前提下，我們可以基於功率最小化原則來優化系統。我們提出了一種幾乎達到最優的優化方法。同用戶隨機共用碼通道相比，基於這個方法的仿真結果顯示系統性能得到了大幅度的提高。基於每個用戶一個時間區間裏面將獨佔一個通道的假設，我們還提出了一系列更簡單的專用時分複用碼通道方案。仿真結果顯示這些簡化的專用方案的優化結果和上面的幾乎最優方案幾乎相同。我們還給出了系統優化的下界。隨著系統容量逐漸增大，我們發現，幾乎最優方案和所有的專用方案都會趨近於系統的優化下界。

由於專用方案簡單而且優化效果也很好，我們考慮了一種“一對一”方法的實際實現。這種方法在每個傳輸的時間段裏面只分配一個碼給一個用戶。我們首先建立了自己的無線通道衰減模型。基於這個模型，我們考慮在保證每個用戶的信噪比的同時最小化傳輸功率。仿真結果顯示系統性能很大程度上倚賴於所用的碼集合。我們測試了 Hadamard 碼和隨機正交碼。仿真結果顯示具有很大選擇多樣性隨機正交碼在最優分配方案下能給出比 Hadamard 好得多得系統性能。在“一對一”方法的實際實現中，我們也考慮了量化和優化頻率對系統的影響。仿真結果顯示我們的“一對一”方法在實際中很有效，它只需要很少的量化過的反饋資訊和比較低的優化頻率就能大大提高系統性能。

Acknowledgments

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I feel a deep sense of gratitude for my parents who formed part of my vision and taught me the good things that really matter in life. The principles they told me still provides a persistent inspiration for my journey in this world.

The chain of my gratitude would be definitely incomplete if I would forget to thank Zhang Li. Without her encouragement, help, and patience, the time I spend in CUHK would not be so delectable.

Contents

Chapter 1 Introduction

There is a rapidly increasing demand for high-speed high-quality wireless access, such as the multimedia data service over wireless channels. Due to the high data rate of the service, a suitable modulation scheme for such broad band communication is needed. However, the traditional single carrier modulation is very vulnerable over the broad band channel, which experiences very severe frequency selective fading. Multicarrier modulation is proposed to address this problem. In multicarrier systems, the wideband channel is divided into some sub-channels whose bandwidth is usually smaller than the fading coherent bandwidth. Therefore, the narrow band signals do not experience frequency selective fading. With this advantage, unlike the single carrier modulation with very complex adaptive equalizer, simple receiver with high performance can be adopted in multicarrier communication systems, which make it very suitable for high-rate wireless mobile communications.

Orthogonal Frequency Division Multiplexing (OFDM) is a matured multicarrier modulation scheme, which transmits parallel data streams over orthogonal sub-carriers. In 1971, Weinstein and Ebert simplified the implementation of OFDM systems with Discrete Fourier Transform (DFT) methods. Since then, OFDM is adopted in many applications such as the IEEE 802.11a and the European HIPERLAN/2 standard for high-speed WLANs. Combining OFDM and CDMA techniques, a novel class of multicarrier communication schemes called multicarrier CDMA emerged in 1993. Much attention has been drawn on this modulation scheme due to its numerous advantages, such as enhancement of robustness against frequency selective fading and high scalability in possible data transmission rate.

1.1 CTDMA for Multicarrier Communication Systems

In a multicarrier communication system, if we consider the downlink, the base station transmits all users' data simultaneously over the sub-carriers to the multiple receivers. In order to separate its data from others', each user must have a distinct signature sequence. That is, orthogonal codes are necessary to be assigned to users as signature sequence for data transmission. If we view these codes as code channels, since different users experience different fading characteristics in wireless mobile environment, different time division of the code channels will give different performance. Therefore, an important problem is to make multiple users share these channels in time domain optimally. This is called code time division multiple access (CTDMA) of the multicarrier communication systems.

Optimization of the CTDMA multicarrier communication system can be considered under different criteria. For the system where the applications are delay sensitive, QoS specifications in terms of data rate requirements can be considered. In our work, we optimize the system by minimizing the total transmission power while the data rate requirement for each user is guaranteed. Due to the nonconvexity of our original formulated optimization problem, we use Taylor expansion to transfer it to a convex programming problem. Based on the matured prime dual interior point algorithm, our near optimal approach can solve the reformulated problem readily. Simulation results show that major performance improvement can be achieved via our near optimal time division approach, comparing with arbitrary time division.

Although the above approach can make multiple users share the code channel near optimally, it take long time for the algorithm to locate the optimal solution of our problem. In order to reduce the computational complexity, by assuming each user will exclusively occupy a code channel in a transmission time slot, we can have simplified ad hoc CTDMA approaches. We consider the case of fully loaded (i.e., the system with same number of users and codes) and non-fully loaded (i.e., the system with more codes than users) multicarrier communication systems respectively. In the fully loaded systems, we develop a one-to-one CTDMA approach based on linear programming or Hungarian method. In this approach, we assign one and only one code to each user and will not adjust the assignment in a whole time slot. We also propose two simplified CTDMA approach for the non-fully loaded systems. Evidently, these ad hoc approaches are all sub-optimal methods, while, simulation results show that they yield performance close to the near optimal approach.

Another important problem is whether these ad hoc approaches are close to the optimal solution of the system optimization. Because it is very difficulty to given the optimal solution directly, we derive a lower bound on the optimal solution based on Taylor formula and Hölder inequality. By evaluating the performance of our near optimal and ad hoc approaches via simulations, we notice that, when the system capacity becomes large, the performance of the near optimal and the ad hoc approaches would converge to the lower bound. This justifies our near optimal approach and ad hoc approaches are all close to the optimal solution, especially in large systems.

Since the one-to-one assignment approach is very close to the lower bound of the system optimization, we consider the implementation of this simple method. In practice point of view, first, we propose a realistic wireless fading model, which considers multipath effect, Doppler effect, shadowing, and large scale fading. Then, we discuss the optimal one-to-one code assignment for multicarrier communication systems. In this case, optimization is achieved by minimize the total transmission power while the signal-to-noise ratio (SNR) for each user will be satisfied. Each user in the system will estimate its channels, quantize the fading coefficients, and then feed back them to the base station. With the feedback information, based on linear programming or Hungarian method, the base station can optimally assign the codes to all users in order to minimize the transmission power. Results show that the performance depends heavily on the set of available codes. To investigate the effects of the codes sets, we use the order of selection diversity to measure the dimension of code set in the optimization. We construct a set of codes called random orthogonal codes with high order of selection diversity based on the QR decomposition. In simulation, we compare the performance of Hadamard codes as well as the random orthogonal codes in optimization.

In practice, we need to consider the effect of quantization on the system performance. Trading off between the performance and complexity, we suggest the appropriate number of bits per fading coefficient for quantization in case of slow fading and fast fading respectively. In our channel model, although the channel is changing over time, while, in a short time slot, each fading coefficient may approximately remain constant. Therefore, we do not need to feed back the coefficients and update the code assignment as frequently as the power control rate. For practical implementation, we also consider the use lower optimization rate in order to reduce system complexity. Simulation results show that the rate of optimization may also be chosen at relatively low frequencies for slow and fast fading respectively.

1.2 Contributions of This Thesis

The main subject of the thesis is the optimization on the CTDMA for multicarrier communication systems. The first part of this thesis provides a review of the concept of multicarrier communications, the OFDM scheme and its application, and the multicarrier CDMA systems. The following introduce the CTDMA scheme for multicarrier communications. Then, we analyze the optimization on the CTDMA for multicarrier systems and a class of solution approaches is proposed. Simulation shows that our approaches perform very well. Finally, we consider the practical implementation of one approach with lower computational complexity.

The main unique contributions of this thesis are the following:

- Near optimal approach for multiple users to CTDMA a multicarrier communication system. We formulate the CTDMA problem as a nonlinear programming problem and transfer it to a convex optimization problem, which can be solved by the prime dual interior point method. The near optimal approach we proposed will converge to the optimality of the original problem as we use more terms to approximate the objective function by Taylor expansion in the reformulation.
- Lower bound on the optimization. Using Taylor expansion on the objective function, we can decompose our original optimization problem into many sub-problems. Based on the Hölder Inequality in LP space $(-\infty,1)$, the lower bound of each sub-problem can be obtained. Finally, we sum all the lower bounds of the sub-problems and get the bound for the original optimization problem.
- Ad Hoc Approaches for CTDMA of multicarrier communication systems. In order to reduce the computational complexity, we propose some simplified ad hoc approaches based on the assumption that one user will exclusive occupy an code channel assigned to it in a hole time slot. We design ad hoc approaches for fully loaded and non-fully loaded systems respectively.

- Practical implementation of the one-to-one ad hoc approach. A realistic wireless fading model is considered in the implementation. We also consider the effects of fading coefficient quantization and the rate of optimization.

1.3 Outline of This Thesis

Chapter 2 gives a survey of the multicarrier communications. After a brief introduction of the concept of multicarrier modulation and its advantages against single carrier modulation, we review the basics of OFDM and the main elements in OFDM transceivers. We also introduce the DFT implement of OFDM systems. Then, we review the three types of multicarrier CDMA communication systems, which emerged in 1993 as a combination of OFDM and CDMA. We discuss the advantages and disadvantages of these three multicarrier CDMA schemes.

Chapter 3 first proposes the multicarrier system model in our work and the concept of CTDMA. Then, we formulate our CTDMA problem as a nonlinear programming. Because of the nonconvexity of our original problem, we transfer it to a convex problem by approximating of the objective function using Taylor formula. The reformulated problem is solved readily by prime dual interior point method. Besides the above near optimal approach, in order to reduce the computational complexity, we also propose some ad hoc approaches for the system optimization. We also derive a lower bound on the optimization. Finally, we evaluate our near optimal approach and simplified ad hoc approaches by simulation. Results show that the ad hoc approaches yield performance close to the near optimal approach. Furthermore, all the approaches converge to the lower bound as the system capacity increase.

Chapter 4 considers the implementation of one ad hoc approach called one-to-one approach. This approach assigns one and only one code to one user in a whole time slot. We also propose a realistic model for wireless mobile fading channel in this chapter. Effects of fading coefficient quantization, feedback, and system optimization rate are also considered in this chapter as realistic issues.

Chapter 2 Multicarrier Communication Systems

The principle of multicarrier communications, which is to transmit data by dividing it into several parallel and interleaved bit streams, and using these to modulate several carriers, has its origin at least thirty yeas ago, when it was applied in the Collins Kineplex system [1]. Recently, much attention has been drawn to high-rate high-quality wireless mobile communications. In hostile wireless channels, such as those in urban area, because of the frequency selective fading, wideband transmission with single carrier modulation may has severe performance degradation due to intersymbol interference (ISI) effect. Multicarrier modulation is therefore considered as an alternative to fulfill such a stringent service requirement. In multicarrier systems, the wideband channel is divided into N narrow band sub-channels that are ISI-free as long as N is sufficiently large. Data steam is also divided into parallel sub-steams. Then, it will be modulated and sent out over these narrow band sub-channels. Because the bandwidth of the sub-channel is usually smaller than the fading coherent bandwidth, the narrow band signals do not experience frequency selective fading. With this advantage, unlike the single carrier modulation with very complex adaptive equalizer, simple receiver with high performance and be adopted in multicarrier communication systems, which make it very suitable for high-rate wireless mobile communications.

2.1 Multicarrier Modulation (MCM) Scheme versus Single Carrier Modulation (SCM) Scheme

The MCM scheme is designed to mitigate ISI by dividing a wideband channel into a number of narrow band sub-channels that are less susceptible to ISI. The rate of the incoming signal R is reduced by a factor of N by increasing the symbol period by N times. In the frequency domain, the bandwidth of the R/N bit steams is N times smaller than the initial bandwidth of the signal. Figure 2.1 shows the R bps data rate signal with period T and baseband bandwidth B, while Figure 2.2, shows one of the R/N bps bit steams with period NT and baseband bandwidth B/N.

Figure 2.1. Time and frequency domain representation of information signal

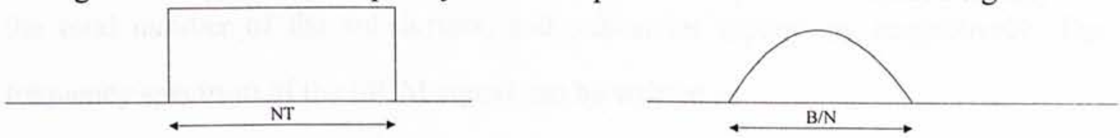

Figure 2.2. Time and frequency domain representation of sub-channel signal

Hence, after passing a serial to parallel converter, the original data stream is divided into N sub-streams and each bit sub-stream is then modulated to one sub-carrier. The modulated signals are summed together and transmitted over the channel. Figure 2.3 shows a typical multicarrier modulation transmitter with a Quadrature Amplitude Modulation (QAM) modulator.

As shown by Figure 2.3, with baseband bandwidth B/N, each sub-channel has a maximum bandwidth of twice its baseband bandwidth, $2B/N$, after pulse shaping and modulation to the passband.

It should be noted that only the in-phase components of the signal are shown in

Figure 2.3. The corresponding quadrature component of the signal is simultaneously modulated by sine wave carriers.

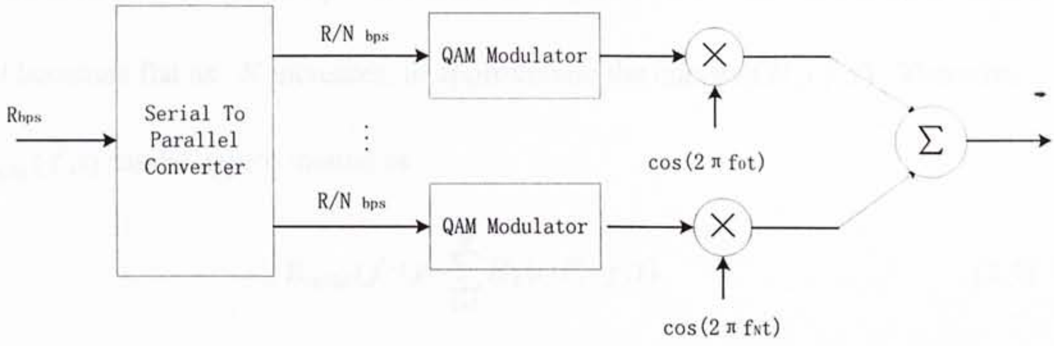

Figure 2.3. Transmitter design of a multicarrier modulation system

Figure 2.4 compares a single carrier (SCM) and a multicarrier modulation (MCM) scheme. Here, B_{SCM} and B_{MCM} denote the bandwidths of transmitted SCM and MCM signals, respectively. For MCM, f_k, $F_k(f;t)$, N and Δf denote the frequency of the kth subcarrier, the frequency spectrum of pulse waveform of the kth subcarrier, the total number of the subcarriers, and subcarrier separation, respectively. The frequency spectrum of the MCM signal can be written as

$$U_{MCM}(f;t)=\sum_{k=1}^{N}F_k(f;t) \tag{2.1}$$

Through a frequency selective fading channel with transfer function $H(f;t)$, the frequency spectra of received SCM and MCM signals are of this form

$$R_{SCM}(f;t)=H(f;t)U_{SCM}(f;t) \tag{2.2}$$

$$\begin{aligned}R_{MCM}(f;t)&=H(f;t)U_{MCM}(f;t)\\&=\sum_{k=1}^{N}H_k(f;t)F_k(f;t)\end{aligned} \tag{2.3}$$

where $U_{SCM}(f;t)$ is the frequency spectrum of transmitted SCM signal and $H_k(f;t)$ is the channel transfer function corresponding to the narrow band B_k, which is the frequency band occupied by the kth subcarrier. When the number of subcarrier is sufficiently large, the amplitude and phase response of $H_k(f;t)$ can be assumed as constant over B_k. As shown in Figure 2.4, we can use a line, which will becomes flat as N increases, to approximate the curve of $H_k(f;t)$. Therefore, $R_{MCM}(f;t)$ can be approximated as

$$R_{MCM}(f;t)=\sum_{k=1}^{N} H_k(t)F_k(f;t) \tag{2.4}$$

where $H_k(t)$ is the complex representation pass loss over the narrow band B_k.

Equation (2.4) and Figure 2.4 clearly show that MCM is effective and robust to combat frequency selective fading. Hence, MCM requires no equalization or at most one-tap equalization for each subcarrier, whereas SCM need complicated adaptive equalization.

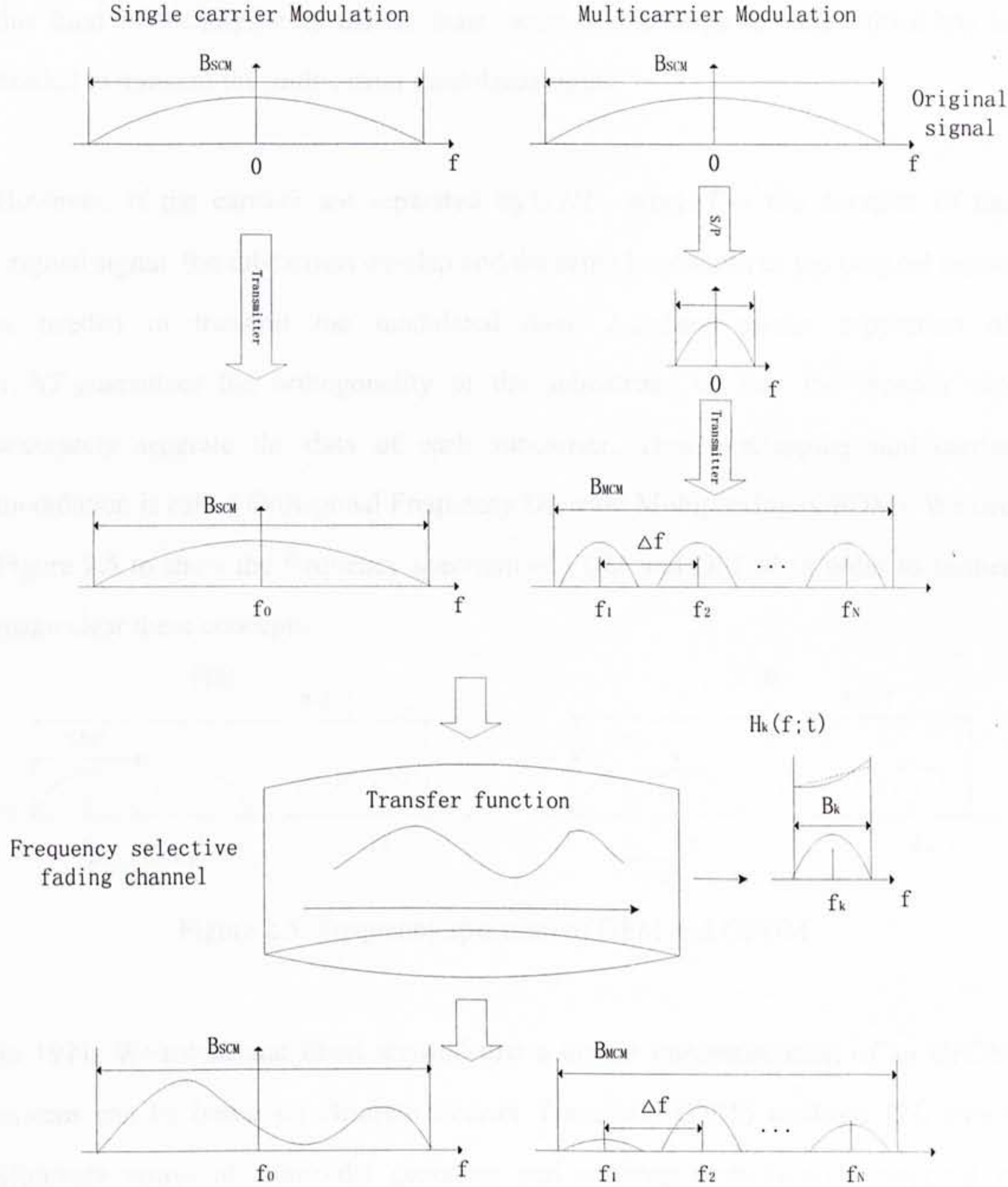

Figure 2.4. Comparison of SCM and MCM

2.2 Orthogonal Frequency Division Multiplexing (OFDM) Systems

If the sub-channel's frequency separation, Δf , is greater than the bandwidth of each subcarrier, $2B/N$, the spectrum of the multicarrier modulated signal will not have overlapped subcarriers, which is called Frequency Division Multiplexing (FDM) or Nonoverlapping Frequency Division Multiplexing (NFDM). The disadvantage of this kind of technigque is that at least twice of the original bandwidth R bps is needed to transmit the multicarrier modulated signal.

However, if the carriers are separated by $1/NT$, where T is the duration of the original signal, the subcarriers overlap and the same bandwidth as the original signal is needed to transmit the modulated data. Adjacent carrier separation of $1/NT$ guarantees the orthogonality of the subcarriers so that the receiver can accurately separate the data of each subcarrier. This overlapping multicarrier modulation is called Orthogonal Frequency Division Multiplexing (OFDM). We use Figure 2.5 to show the frequency spectrum of FDM and OFDM in order to further make clear these concepts.

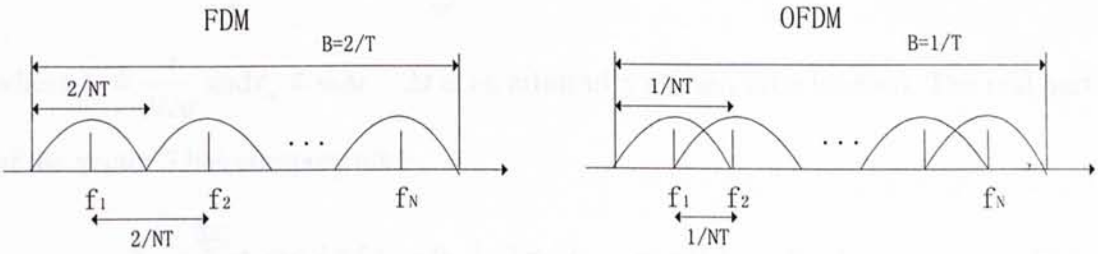

Figure 2.5. Frequency spectrum of DFM and OFDM

In 1971, Weinstein and Ebert showed that a digital implementation of an OFDM system can be based on Discrete Fourier Transform (DFT) methods [2], which eliminate arrays of sinusoidal generator and coherent demodulation required in parallel data systems and make the implementation of the technology cost effective. However, it still took more than a decade from that time until OFDM start to attract broad interest, boosted by the demand for broadband communication systems and the vast development in digital signal processing technology. Before we investigate the utilization of OFDM in today's communication systems, let's review how DFT can be used to implement OFDM.

A block diagram of the OFDM transmitter implemented by DFT is given in Figure

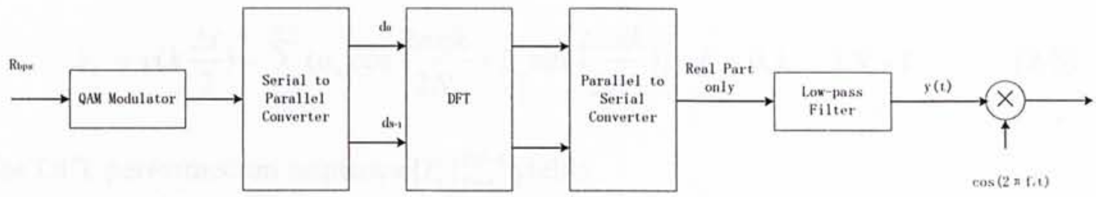

Figure 2.6. Block diagram of OFDM transmitter with DFT technology

2.6. Consider a data sequence $[d_0, d_1, ..., d_{N-1}]$, where each d_i is a complex number $d_n = a + jb_n$. If we perform a DFT on the vector $\{d_n\}_{n=0}^{N-1}$, the result is a vector $S = \{S_0, S_1, \cdots, S_{N-1}\}$ of N complex numbers with the form as follows

$$S_m = \sum_{n=0}^{N-1} d_n e^{-j(2\pi nm/N)} = \sum_{n=0}^{N-1} d_n e^{-j2\pi f_n t_m}, \quad m = 0, 1, \cdots, N-1 \tag{2.5}$$

where $f_n \triangleq \frac{n}{N\Delta t}$ and $t_m \triangleq m\Delta t$. Δt is an arbitrarily chosen time interval. The real part of the vector S has components

$$Y_m = \sum_{n=0}^{N-1} a_n \cos 2\pi f_n t_m + b_n \sin 2\pi f_n t_m, \quad m = 0, 1, \cdots, N-1 \tag{2.6}$$

If these components are applied to a low-pass filter at time interval $m\Delta t$, a signal is obtained that closely approximates the original OFDM signal

$$y(t) = \sum_{n=0}^{N-1} a_n \cos 2\pi f_n t + b_n \sin 2\pi f_n t, \quad 0 \le t \le N\Delta t \tag{2.7}$$

Demodulation at the receiver is also carried out via the DFT of a vector of samples of the received signal. Figure 2.7 shows the structure of a conventional OFDM receiver.

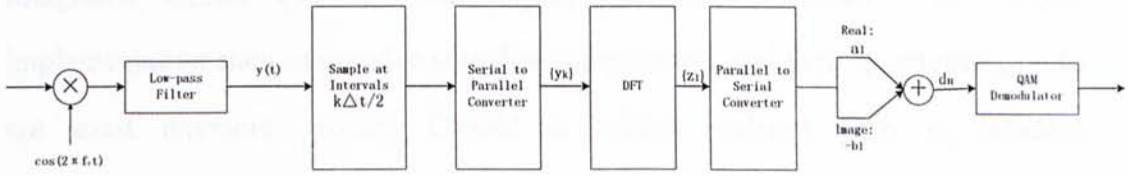

Figure 2.7. Block diagram of OFDM receiver with DFT technology

Because only the real part of the original OFDM signal is transmitted, it is necessary to sample twice as fast as expected, i.e., at intervals $\Delta t / 2$. When there is no channel distortion, the total $2N$ samples of the received signal is as follows

$$Y_k = y(k\frac{\Delta t}{2}) = \sum_{n=0}^{N-1}(a_n \cos\frac{2\pi nk}{2N} + b_n \sin\frac{2\pi nk}{2N}), \quad k = 0,1,\cdots 2N-1 \tag{2.8}$$

The DFT performed on sequence $\{Y_k\}_{k=0}^{2N-1}$ yields

$$z_l = \frac{1}{2N}\sum_{k=0}^{2N-1} Y_k e^{-j(2\pi lk/2N)} = \begin{cases} a_0, & l = 0 \\ \dfrac{a_l - jb_l}{2}, & l = 1,2,\cdots N-1 \\ irrelevant, & l > N-1, \end{cases} \tag{2.9}$$

where the equality

$$\frac{1}{2N}\sum_{k=0}^{2N-1} e^{j(2\pi mk/2N)} = \begin{cases} 1, & m = 0, \pm 2N, \pm 4N, \cdots \\ 0, & otherwise \end{cases} \tag{2.10}$$

has been employed. As shown by equation (2.9), the original data a_l and b_l are available as the real and image part of z_l. Hence, the original d_n can be recovered. After passing the QAM demodulator, we can get the source symbols with data rate of R bps.

Now, we can see that OFDM communication systems can be readily built up with the DFT technology. Furthermore, if the number of subcarriers N is power of 2, many Fast Fourier Transform (FFT) algorithms can be adopted to perform the DFT transform, which will further reduce the system complexity.

Due to recent advances of digital signal Processing (DSP) and Very Large Scale Integrated circuit (VLSI) technologies, the initial obstacles of OFDM implementation such as massive complex computation, and high speed memory do not exist anymore. Today, OFDM is widely utilized both in wireless communications and for cable based data transmission. In the latter field, the technology is normally called Discrete Multitone (DMT) and applied in the new xDSL standards like Asymmetric DSL (ADSL). In wireless transmission, ODFM was adopted by the IEEE 802.11a and the European HIPERLAN/2 standard for high-speed WLANs. Moreover, OFDM was chosen for starting up terrestrial digital broadcasting services like the Digital Audio Broadcasting (DAB) systems and Digital Video Broadcasting (DVB) projects.

2.3 Multicarrier CDMA

Code Division Multiple Access (CDMA) is a multiplexing technique where a number of users simultaneously and asynchronously access a channel by spreading each user's information with distinct signature sequences. In 1993, an epoch of CDMA application, there types of new multiple access schemes based on a combination of CDMA and OFDM were proposed. N.Yee, J-P Linnartz and G. Fettweis [3], K. Fazel and L. Papke [4], and A. Chouly, A. Brajal and S. Jourdan [5] proposed Multicarrier (MC-) CDMA. V. M. Dasilva and E. S. Sousa [6] proposed Multicarrier DS-CDMA. L. Vandendorpe [7] proposed Multitone (MT-) CDMA. Soon, Multicarrier CDMA becomes a very hot topic and lots of research works have done on it. In this section, we will review these three types of Multicarrier CDMA schemes and discuss their advantages and disadvantages in terms of the transmitter and receiver structures.

2.3.1 MC-CDMA

Unlike the conventional CDMA that spreads the original data stream using the pre-assigned spreading code in the time, the MC-CDMA transmitter spreads user's data stream over different subcarriers using a given spreading code in the frequency domain. In other word, each subcarrier is modulated by a chip of the user's spreading code. In [8], K. Fazel and G. Fettweis have shown that spreading codes like the Hadamard codes are optimum in maintaining orthogonality between subcarriers and reducing intermodulation in nonlinear amplifiers.

We consider a system of totally M users. We define the vector $\mathbf{C}_m = [C_{m,1}, C_{m,2}, \cdots, C_{m,N}]^T$ as the spreading code for the m th user. By choosing M orthogonal vectors, we can construct the MC-CDMA transmitter for totally M users. Without loss generosity, we depicted the structure of the MC-CDMA transmitter of the m th user in Figure 2.8. All the users signal are summed together, modulated the carrier frequency f_c, and sent out through wireless mobile channel. If we assume the channel is distortion-free, without loss of generosity, we consider use the receiver as shown by Figure 2.9 to receive and recover the data stream of the m th user. The transmitter and receiver for other users can be readily obtained by adopting their own spreading code, i.e., appropriately change the subscripts of the spreading code in Figure 2.8 and Figure 2.9.

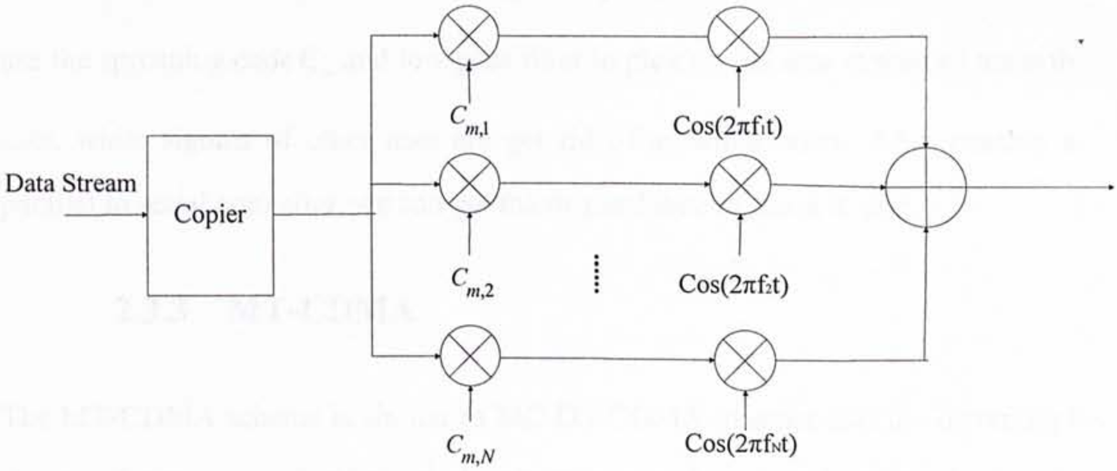

Figure 2.8. Block diagram of the MC-CDMA transmitter for one user

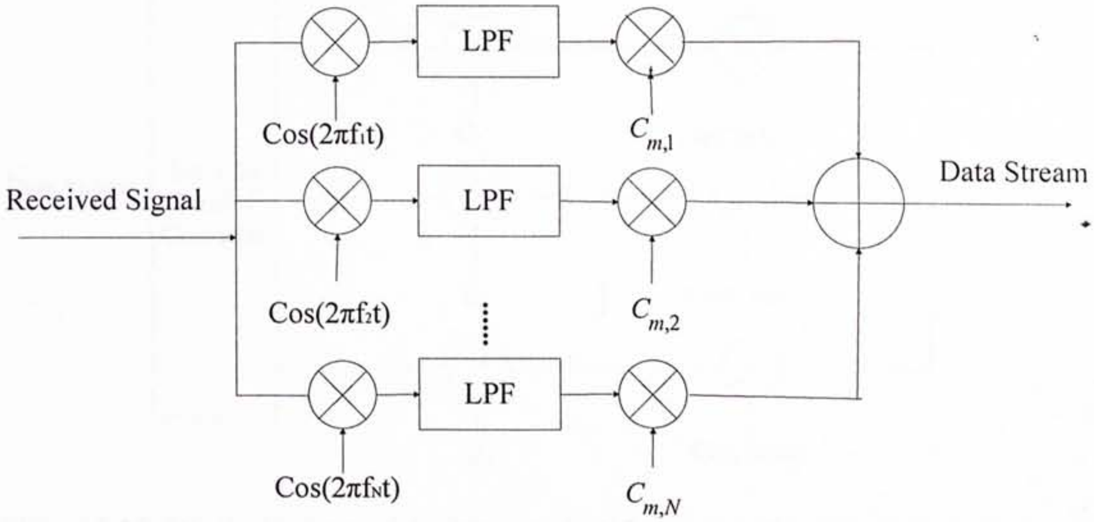

Figure 2.9. Block diagram of the MC-CDMA receiver for one user

2.3.2 MC-DS-CDMA

The MC-DS-CDMA transmitter spreads the serial/parallel converted data stream using a given spreading code in the time domain so that the resulting spectrum of each subcarrier can satisfy the orthogonality condition with the minimum frequency separation [6]. We consider the m th user and assume $\mathbf{C}_m = [C_{m,1}, C_{m,2}, \cdots, C_{m,N}]^T$ as its spreading code. As shown by Figure 2.10, the symbols modulated on the N carriers are summed together before being transmitted over the channel. We use the receiver shown by Figure 2.11 for the data reception of the m th user. If we assume the wireless channel has no distortion, all users' data will maintain their orthogonality. Therefore, at the receiver end, we use the spreading code $\mathbf{C}_m$ and low-pass filter to pick up the data stream of the m th user, while signals of other user are get rid of as white noise. After passing a parallel to serial converter, we can get the original data of the m th user.

2.3.3 MT-CDMA

The MT-CDMA scheme is similar to MC-DS-CDMA in sense that the incoming bit stream of one user is divided into N sub-streams and every sub-stream is spread in

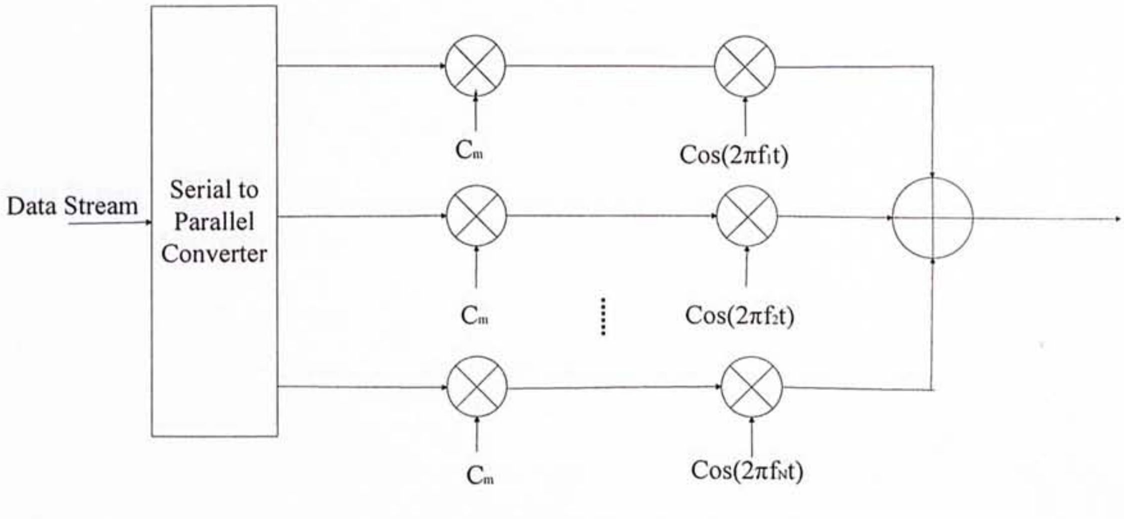

Figure 2.10. Block diagram of the MC-DS-CDMA transmitter for one user

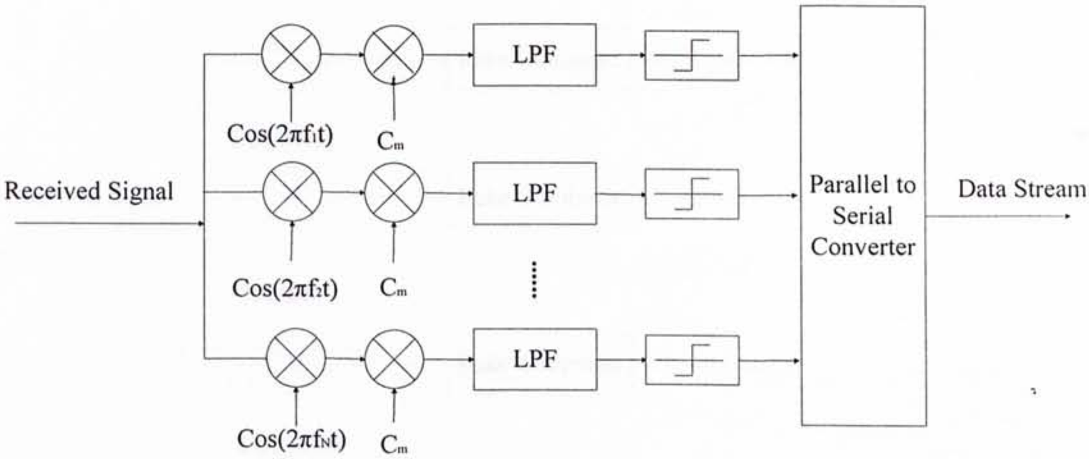

Figure 2.11. Block diagram of the MC-DS-CDMA receiver for one user

time domain with the same signature sequence pre-assigned to that particular user. Unlike MC-DS-CDMA, MT-CDMA uses longer spreading codes in proportion to the number of subcarriers. After spreading, the data on each subcarrier are no longer orthogonal. Therefore, when we consider the design of receiver, Rake Combiner with the same structure as DS-CDMA Rake Receiver must be used to receive the data stream of each subcarrier. Without loss of generosity, Figure 2.12 and Figure 2.13 show the structure of MT-CDMA transmitter and receiver of the m th user, respectively. Other users' transmitter and receiver can be readily designed by appropriately changing the subscripts.

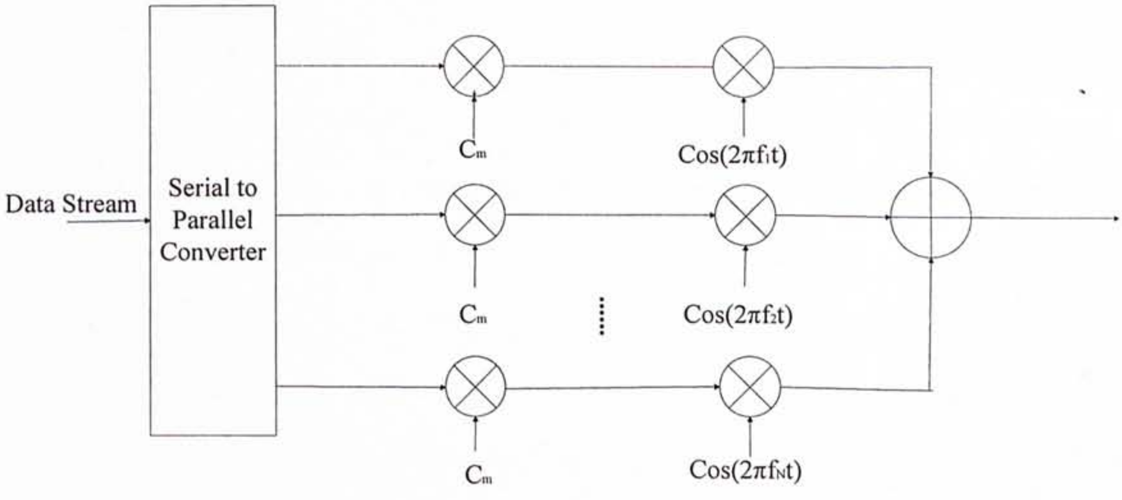

Figure 2.12. Block diagram of the MT-CDMA transmitter for one user

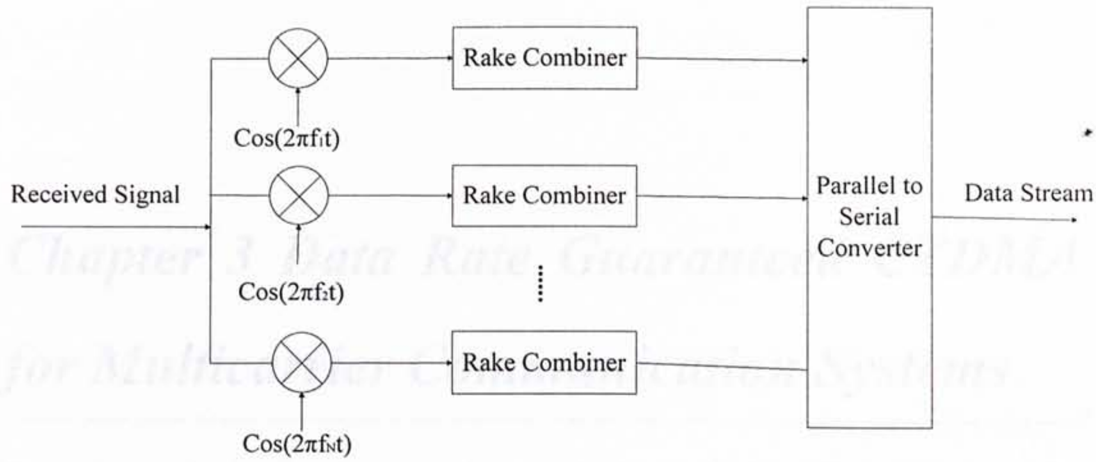

Figure 2.13. Block diagram of the MT-CDMA receiver for one user

Chapter 3 Data Rate Guaranteed CTDMA for Multicarrier Communication Systems

Recently, much attention has been drawn to high-rate high-quality wireless mobile communications. Multicarrier modulation is proposed to deliver such a stringent service requirement and gained much attention in recent years. On the downlink of a multicarrier communication system, orthogonal codes are usually assigned to users for data transmission. We can view these codes as code channels. In a wireless communication system, since different users experience different fading characteristics, different time division of the code channels will give different performance. Therefore, an important problem is to make multiple users share these channels in time domain optimally. This is called code time division multiple access (CTDMA) of the multicarrier communication systems. We will discuss the details of CTDMA is Section 3.1

Optimal CTDMA approach can be considered under different criteria. For a system where the applications are delay sensitive, we optimize it by minimizing the total transmission power while the data rate requirement for each user is guaranteed. After we introduce basic concepts and methods in mathematically programming in Section 3.2, based on interior point algorithm for nonlinear convex programming, in Section 3.3 we propose our near optimal time division approach, which can offer major performance improvement, comparing with arbitrary time division.

In order to reduce computational complexity, in Section 3.4, we propose a family of simplified ad hoc time division approaches. The commonness of these approaches is that we assume each user will exclusively occupy a code channel in a time slot. In other words, as long as a code is assigned to one user, it can not be assigned to other user in the current time slot. We consider the case of fully loaded and non-fully loaded multicarrier communication systems respectively. In the fully loaded systems, i.e., systems with same number of users and codes, we develop a one-to-one CTDMA approach based on linear programming or Hungarian method. In the non-fully loaded systems, i.e., systems with more codes than users, two assignment approaches are developed. Simulation results show that the ad hoc approaches yield performance close to the near optimal approach. Therefore, these practical methods with low computational complexity can be implemented readily in multicarrier communication systems.

In Section 3.5, based on Taylor formula and Hölder inequality, we derive a lower bound on the optimal solution. In Section 3.6, we evaluate the performance of our near optimal and ad hoc approaches by simulations. We also compare the performance of our approaches with arbitrary time division approach and the lower bound. We notice that when the system capacity becomes large, the performance of the near optimal and the ad hoc approaches would converge to the lower bound.

3.1 Code Time Division Multiple Access (CTDMA)

In this section, first, we describe the model of the MC-CDMA system. We assume there are N carrier frequencies and N simultaneous users in the system. We consider the downlink. The base station uses the N carrier frequencies to create N orthogonal transmission channels as follows: To transmit a narrowband signal $S_m(t)$ to a user, the base station sends the signal with the following complex representation

$$\sqrt{p_m}\sum_{n=1}^{N} C_{m,n} S_m(t) e^{jw_n t} \tag{3.1}$$

where p_m is the power of the signal, and $C_{m,n}$ is the gain factor of the nth carrier. The structure of the MC transmitter for one signal is depicted in Figure 3.1. We define the vector

$$\mathbf{C}_m = [C_{m,1}, C_{m,2}, \ldots, C_{m,N}]^T \tag{3.2}$$

By choosing N orthogonal vectors (i.e., $\mathbf{C}_m$ for $m = 1,\ldots,N$), N orthogonal transmission channels can be formed. We assume that the norm of each vector is normalized to one unit. As a special case, if the orthogonal vectors are the unit coordinate vectors, then the multicarrier system reduces to a FDMA system.

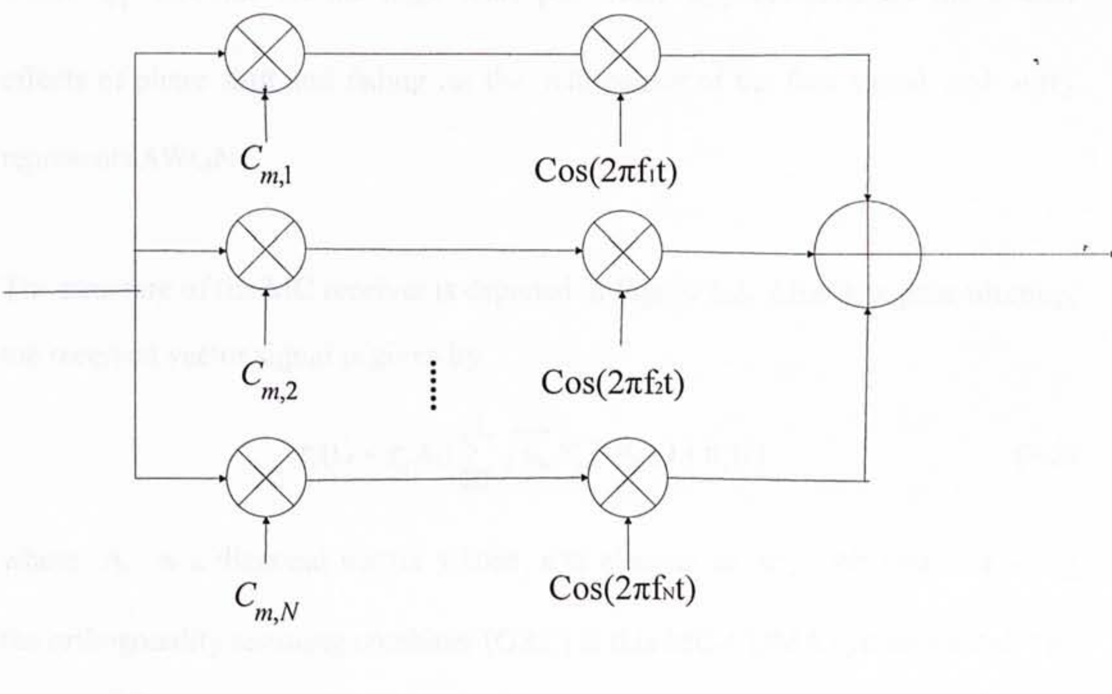

Figure 3.1. Block diagram of the MC transmitter for one user

The base station uses N channels to serve up to N users. The total transmitted signal in any time slot is of the form

$$\sum_{m=1}^{N}\sqrt{p_m}\sum_{n=1}^{N} C_{m,n} S_m(t) e^{jw_n t} \tag{3.3}$$

We assume that the signals $S_m(t)$ are bandlimited, and are normalized to unit power. By suitably chosen N carrier frequencies, we can assume that signals on different carriers are orthogonal and do not interfere with others. We assume that each carrier undergoes independent frequency non-selective Rayleigh fading. We can also assume the presence of addictive white Gaussian Noise (AWGN).

Without loss of generosity, we consider the receiver for the first signal. The received signal in the complex analytic representation is given by

$$r_1(t) = g_1 \sum_{m=1}^{N} \sqrt{p_m} \sum_{n=1}^{N} C_{m,n} S_m(t) e^{j w_n t} \alpha_{1,n} + n_1(t) \tag{3.4}$$

where g_1 accounts for the large scale path loss, $\alpha_{1,n}$ accounts for the overall effects of phase shift and fading for the nth carrier of the first signal, and $n_1(t)$ represents AWGN.

The structure of the MC receiver is depicted in Figure 3.2. After low pass filtering, the received vector signal is given by

$$\mathbf{r}_1(t) = g_1 \mathbf{A}_1 (\sum_{m=1}^{N} \sqrt{p_m} S_m(t) \mathbf{C}_m) + \mathbf{n}_1(t) \tag{3.5}$$

where $\mathbf{A}_1$ is a diagonal matrix whose nth element is $\alpha_{1,n}$. We consider using the orthogonality restoring combiner (ORC) is this MC-CDMA system model. The output of the receiver is therefore given by

$$z_1(t) = g_1 \sqrt{p_1} S_1(t) + \mathbf{C}_1^H \mathbf{A}_1^{-1} \mathbf{n}_1(t) \tag{3.6}$$

The SNR is given by

$$\gamma_1 = \frac{g_1^2 p_1}{\sigma^2 \mathbf{C}_1^H \mathbf{A}_1^{-1} \mathbf{A}_1^{-1H} \mathbf{C}_1} \tag{3.7}$$

where σ^2 is the variance of the AWGN contribution.

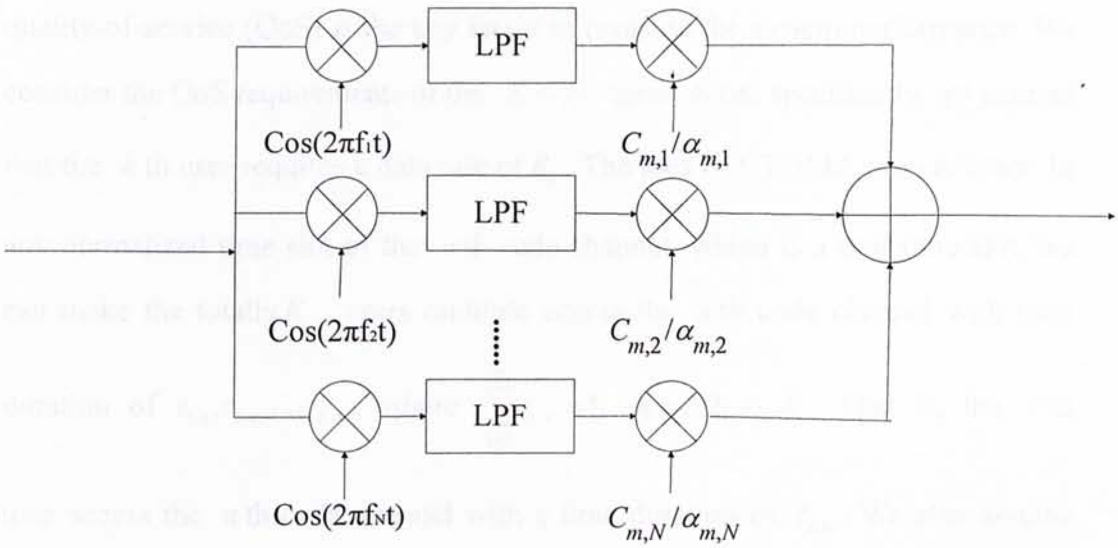

Figure 3.2. Block diagram of the MC receiver for one user

With suitable error control coding, the data rate can reach

$$r_1 = B\log_2(1+\gamma_1) \tag{3.8}$$

Therefore, the transmission power to guarantee data rate requirement r_1 of the first user is given by

$$p_1 = \frac{(2^{r_1/B}-1)\sigma^2 \mathbf{C}_1^H \mathbf{A}_1^{-1} \mathbf{A}_1^{-1H} \mathbf{C}_1}{g_1^2} \tag{3.9}$$

The corresponding quantities for the other users can be obtained by appropriately changing the subscripts.

We assume there are totally $K \le N$ users in a multicarrier communication system with totally N orthogonal codes. If we view the codes as code channels, as we have mentioned, in time domain, all K users need to share the N code channels. In other words, K users will time division multiple access the N code channels. Then, an important problem is how to make the multiple users time-share the code channels optimally.

In high-rate high-quality wireless mobile communication systems, quality-of-service (QoS) is the key factor to measure the system performance. We consider the QoS requirements of the $K \le N$ users. More specifically, we assume that the k th user requires a data rate of R_k. The idea of CTDMA is as follows: In any normalized time slot of the n th code channel, which is a unit time slot, we can make the totally K users multiple access the n th code channel with time duration of $t_{1,n}, t_{2,n}, \ldots, t_{K,n}$, where $\sum_{k=1}^{K} t_{k,n} = 1, \quad n = 1, 2, \cdots, N$. That is, the k th user access the n th code channel with a time duration of $t_{k,n}$. We also assume that, in the time interval $t_{k,n}$, the n th code channel serves the k th user with a data rate $r_{k,n}$. Figure 3.3 shows the CTDMA scheme of a K users N codes multicarrier system.

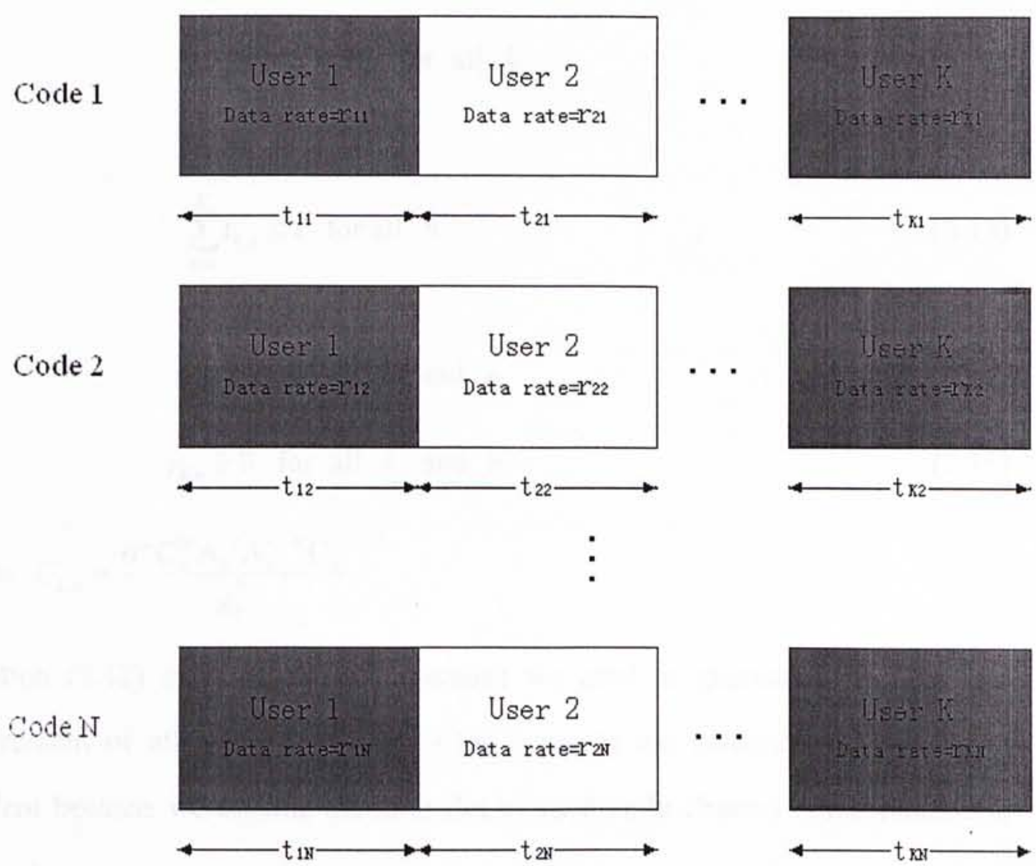

Figure 3.3. CTDMA scheme of multicarrer communication systems

Therefore, our problem is to find the optimal time division and data rate control variables $t_{k,n}$ and $r_{k,n}$, for all k and n, to minimize the total transmission power while the data rate requirements of all the users is guaranteed. When the nth code channel is occupied by the kth user with a data rate of $r_{k,n}$, the power required is of this form

$$p_{k,n}(r_{k,n}) = \frac{(2^{r_{k,n}}-1)\sigma^2 \mathbf{C}_n^H \mathbf{A}_k^{-1} \mathbf{A}_k^{-1H} \mathbf{C}_n}{g_k^2} \tag{3.10}$$

Based on transmission power minimization criteria, we can formulate our problem as follows

$$\text{Minimize } P = \sum_{k=1}^{K}\sum_{n=1}^{N} p_{k,n}(r_{k,n}) t_{k,n} = \sum_{k=1}^{K}\sum_{n=1}^{N} t_{k,n}(2^{r_{k,n}/B}-1)C_{k,n} \tag{3.11}$$

subject to

$$\sum_{n=1}^{N} r_{k,n} t_{k,n} = R_k \quad \text{for all } k \tag{3.12}$$

$$\sum_{k=1}^{K} t_{k,n} \le 1 \quad \text{for all } n \tag{3.13}$$

$$t_{k,n} \ge 0 \quad \text{for all } k \text{ and } n \tag{3.14}$$

$$r_{k,n} \ge 0 \quad \text{for all } k \text{ and } n \tag{3.15}$$

where $C_{k,n} = \dfrac{\sigma^2 \mathbf{C}_n^H \mathbf{A}_k^{-1} \mathbf{A}_k^{-1H} \mathbf{C}_n}{g_k^2}$.

Equation (3.12) must be satisfied because we need to guarantee the data rate requirement of all users. Equation (3.13) becomes the constraint of the above problem because we assume the time slot of each code channel is normalized to one unit.

Generally speaking, the above problem is a nonconvex nonlinear programming problem because of the nonlinearity of equality constrains. Therefore, the optimal solution, i.e., the global minimum, is very difficult to obtain. Hence, the doable approach to solve this problem is to transform it to a convex nonlinear programming problem, whose global minimum can located with many methods, such as the interior point algorithm. Before discussing the approaches to solve our problem, we introduce some concepts concerning the mathematical programming.

3.2 Mathematical Programming

A mathematical programming problem, or optimization problem, has the following form

$$\text{Minimize } f_0(x) \tag{3.16}$$

$$\text{subject to } f_i(x) \leq 0, \quad i = 1, \cdots, m \tag{3.17}$$

$$h_i(x) = 0, \quad i = 1, \cdots, p \tag{3.18}$$

where the vector $x = [x_1, x_2, \cdots, x_n]$ is the *optimization variable* of the problem, the function $f_0 : R^n \to R$ is called the *objective function* or *cost function*, the functions $f_i : R^n \to R, \quad i = 1, \cdots, m,$ are called the *inequality constraint functions*, and the corresponding inequality $f_i(x) \leq 0, \quad i = 1, \cdots, m$ are called *inequality constraints*. The functions $h_i : R^n \to R, \quad i = 1, \cdots, p,$ are called the *equality constraint functions*, and the corresponding equality $h_i(x) \leq 0, \quad i = 1, \cdots, p,$ are called *equality constraints*. If there are no constraints (i.e. $m = p = 0$) the problem is said to be *unconstrained*. A vector x^* is called *optimal solution* of the above programming problem, if it has the smallest objective value among all vectors that satisfy the constraints: for any z with $f_1(z) \leq 0, \cdots, f_m(z) \leq 0$ and $h_1(z) = 0, \cdots, h_p(z) = 0$. we have $f_0(z) \geq f_0(x^*)$.

Generally, we consider families or classes of optimization problem, which are characterized by different forms of objective and constrains functions. One important example is the *linear programming* problem, whose objective and constraint functions $f_0, \cdots, f_m, h_0, \cdots, h_p$ are linear, i.e. satisfy $f_i(\alpha x + \beta y) = \alpha f_i(x) + \beta f_i(y)$ for all $x, y \in R^n$ and all $\alpha, \beta \in R$. If the optimization problem is not linear, it is called a *nonlinear programming* problem.

3.2.1 Nonlinear Programming

Usually, nonlinear programming is the term used to describe an optimization problem where the objective or constraint functions are not linear, but not known to be convex. Sadly, there are no effective methods for solving the general nonlinear programming problem. Even simple looking problems with as few as ten variables can be extremely challenging, while problems with a few hundreds of variables can be intractable. Methods for the general nonlinear programming problem therefore take several different approaches, each of which involves some compromise.

One class of approaches is called *local optimization*. In this kind of approaches, the compromise is to give up seeking the optimal x, which minimizes the objective over all feasible points. Instead we seek a point that is only locally optimal, which means that it minimizes the objective function among feasible points that are near it, but is not guaranteed to have a lower objective value than all other feasible points. A large fraction of the research on general nonlinear programming has focused on methods for local optimization, which as a consequence are well developed.

Local optimization methods can be fast, can handle large-scale problems, and are widely applicable, since they only require differentiability of the objective and constraint functions. As a result, local optimization methods are widely used in applications where there is value in finding a good point, if not the very best. In an engineering design application, for example, local optimization can be used to improve the performance of a design originally obtained by manual, or other, design methods.

There are several disadvantages of local optimization methods, beyond (possibly) not finding the true, globally optimal solution. The methods require an initial guess for the optimization variable. This initial guess or starting point is critical, and can greatly affect the objective value of the local solution obtained. Little information is provided about how far from (globally) optimal the local solution is. Local optimization methods are often sensitive to algorithm parameter values, which may need to be adjusted for a particular problem, or family of problems.

Another family of approaches for nonlinear programming is *global optimization* approach. In global optimization, the true global solution of the optimization problem is found; the compromise is efficiency. The worst-case complexity of global optimization methods grows exponentially with the problem sizes, i.e., the dimension of the x vector and the number of constraints; the hope is that in practice, for the particular problem instances encountered, the method is far faster. While this favorable situation does occur, it is not typical. Even small problems, with a few tens of variables, can take a very long time (e.g., hours or days) to solve. Usually, global optimization is used for problems with a small number of variables, where computing time is not critical, and the value of finding the true global solution is very high.

3.2.2 Convex Programming

In point of view of convexity, we can also divide the programming problems into two classes, *convex* and *nonconvex* programming. A convex optimization problem is one of the form

$$\text{Minimize } f_0(x) \tag{3.19}$$

$$\text{subject to } f_i(x) \le 0, \quad i = 1, \cdots, m \tag{3.20}$$

$$a_i^T x = b_i, \quad i = 1, \cdots, p \tag{3.21}$$

where $f_0, \cdots, f_m$ are convex functions, i.e., satisfy

$$f_i(\alpha x + \beta y) \le \alpha f_i(x) + \beta f_i(y) \tag{3.22}$$

for all $x, y \in R^n$ and all $\alpha, \beta \in R$ with $\alpha + \beta = 1, \alpha \ge 0, \beta \ge 0$

Comparing with the general standard form problem, the convex problem has three additional requirements:

- The objective function must be convex,
- The inequality constraint functions must be convex,
- The equality constraint functions $h_i(x) = a_i^T x - b_i$ must be affine.

There is in general no analytical formula for the solution of convex optimization problems, but (as with linear programming problems) there are very effective methods for solving them. Interior-point methods work very well in practice, and in some cases can be proved to solve the problem to a specified accuracy with a number of operations that does not exceed a polynomial of the problem dimensions.

Using convex optimization is, at least conceptually, very much like using linear programming. If we can formulate a problem as a convex optimization problem, then we can solve it efficiently, just as we can solve a linear problem efficiently. With only a bit of exaggeration, we can say that, if you formulate a practical problem as a convex optimization problem, then you have solved the original problem.

A very important property of convex programming problem is that any local optimal solution is also the global optimal solution. To see this, suppose that x is locally optimal for a convex programming problem, i.e., x is feasible and

$$f_0(x) = \inf\{f_0(z) \mid z \text{ } feasible, \|z - x\|_2 \leq R\}, \tag{3.23}$$

for some $R > 0$. Now, if we suppose that x is not globally optimal, that is, there is another feasible solution y such that $f_0(y) < f_0(x)$. Evidently $\|y - x\|_2 > R$, since otherwise $f_0(x) < f_0(y)$. We consider the point z of the form

$$z = (1-\theta)x + \theta y, \qquad \theta = \frac{R}{2\|y - x\|_2} \tag{3.24}$$

Then we have $\|z - x\|_2 = R/2 < R$. By the convexity of $f_0(x)$, we have

$$f_0(z) \leq (1-\theta) f_0(x) + \theta f_0(y) < f_0(x) \tag{3.25}$$

which contradicts (3.23). Therefore, there is no feasible y with $f_0(y) < f_0(x)$, that is, x is globally optimal. Hence, we can conclude that the local optimal solution of a convex nonlinear programming problem is also the global optimal.

3.3 Near Optimal CTDMA Approach

Unfortunately, as mentioned in section 3.1, our original optimization problem is a nonconvex problem because it does not satisfy the third requirement of convex optimization since the equality constraint function (3.12) is not affine. Therefore, the original problem becomes intractable to locate its global optimal solution. Furthermore, when the system capacity becomes large, i.e., there are many users in the system, even if some methods can solve the original problem, they must be very inefficient and time-consuming. Therefore, we need to reformulate the original nonconvex problem to a form of convex programming, which can be solved efficiently by the well developed method, the interior point algorithm.

3.3.1 Problem Reformulation

First, we use Taylor's expansion formula to reformulate the objective function as follows

$$P=\sum_{k=1}^{K}\sum_{n=1}^{N}t_{k,n}(2^{r_{k,n}/B}-1)C_{k,n}=\sum_{k=1}^{K}\sum_{n=1}^{N}t_{k,n}C_{k,n}\sum_{m=1}^{+\infty}\frac{1}{m!}(\frac{r_{k,n}\ln 2}{B})^{m}, \tag{3.26}$$

according to

$$2^{\alpha}-1=e^{\alpha\ln 2}-1=\alpha\ln 2+\frac{(\alpha\ln 2)^{2}}{2!}+\cdots+\frac{(\alpha\ln 2)^{m}}{m!}+\cdots \tag{3.27}$$

We define $x_{k,n}=\frac{t_{k,n}r_{k,n}\ln 2}{B}$ and $\hat{R}_k=\frac{R_k\ln 2}{B}$. Then, we can reformulate our original problem as follows

$$\text{Minimize}\quad \sum_{m=1}^{+\infty}\frac{1}{m!}\sum_{k=1}^{K}\sum_{n=1}^{N}C_{k,n}t_{k,n}^{1-m}x^{m}{}_{k,n} \tag{3.28}$$

subject to

$$\sum_{n=1}^{N}x_{k,n}=\hat{R}_k \quad \text{for all } k \tag{3.29}$$

$$\sum_{k=1}^{K}t_{k,n}\le 1 \quad \text{for all } n \tag{3.30}$$

$$t_{k,n}\ge 0 \quad \text{for all } k \text{ and } n \tag{3.31}$$

$$x_{k,n}\ge 0 \quad \text{for all } k \text{ and } n \tag{3.32}$$

Equation (3.28) is a summation of infinite term. Because of the property of Taylor expansion, the high order terms become ignorable. Hence, by choosing sufficiently large M, we can have a very good approximation of the original problem by the further reformulate the original problem as follows

$$\text{Minimize}\quad \sum_{m=1}^{M}\frac{1}{m!}\sum_{k=1}^{K}\sum_{n=1}^{N}C_{k,n}t_{k,n}^{1-m}x^{m}{}_{k,n} \tag{3.33}$$

subject to

$$\sum_{n=1}^{N} x_{k,n} = \hat{R}_k \quad \text{for all } k \tag{3.34}$$

$$\sum_{k=1}^{K} t_{k,n} \leq 1 \quad \text{for all } n \tag{3.35}$$

$$t_{k,n} \geq 0 \quad \text{for all } k \text{ and } n \tag{3.36}$$

$$x_{k,n} \geq 0 \quad \text{for all } k \text{ and } n \tag{3.37}$$

Before we prove this reformulated problem is a standard form of convex programming, we recall the requirements of the form of convex optimization problem and list them in the following:

- The objective function must be convex,
- The inequality constraint functions must be convex,
- The equality constraint functions $h_i(x) = a_i^T x - b_i$ must be affine.

We can easily check that, the inequality constraint functions, $\sum_{k=1}^{K} t_{k,n} \leq 1$ for all n, are linear functions. Because linear function is a special case of convex function, the second requirement is guaranteed. In the above problem, the equality constraint functions, $\sum_{n=1}^{N} x_{k,n} = \hat{R}_k$ for all k, is also linear. Therefore, they are affine and satisfy the third requirement. Now, we focus on the first requirement and prove the objective function is a convex function.

Lemma 1

Given $t > 0, x > 0 \; and \; m \geq 1$, function $h(t,x) = t^{1-m} x^m$ is a convex function.

Proof:

The *second order conditions* of convex function tells us that a function f is convex if and only if its Hessian is positive semidefinite. The Hessian of function $h(t,x) = t^{1-m} x^m$ is

$$\begin{bmatrix} \frac{\partial^2 h}{\partial x^2} & \frac{\partial^2 h}{\partial x \partial t} \\ \frac{\partial^2 h}{\partial t \partial x} & \frac{\partial^2 h}{\partial t^2} \end{bmatrix} = \begin{bmatrix} m(m-1)t^{1-m}x^{m-2} & m(1-m)t^{-m}x^{m-1} \\ m(1-m)t^{-m}x^{m-1} & -m(1-m)t^{-m-1}x^{m} \end{bmatrix}$$
$$= m(m-1)t^{-m}x^{m-1} \begin{bmatrix} \frac{t}{x} & -1 \\ -1 & \frac{x}{t} \end{bmatrix} \tag{3.38}$$

Then, we check whether the above matrix is semidefinite. Without difficulty, we can calculate the two eigenvalues of the above matrix is 0 and $\frac{t}{x}+\frac{x}{t}$, with corresponding eigenvectors of $[\frac{x}{t},1]^T$ and $[-\frac{t}{x},1]^T$,respectively. Because the two eigenvalues are both nonnegative, the Hessian of h is positive semidefinite. Therefore, we prove *Second order conditions* of convex are satisfied for function h. Hence, Lemma 1 is proved.

Lemma 2

Given $h(t_{1,1},\cdots,t_{K,N},x_{1,1},\cdots,x_{K,N}) = \sum_{k=1}^{K}\sum_{n=1}^{N} C_{k,n} t_{k,n}^{1-m} x^{m}{}_{k,n}$, $h(t_{1,1},\cdots,t_{K,N},x_{1,1},\cdots,x_{K,N})$ is a convex function.

Proof:

We use $\vec{t}$ and $\vec{x}$ to denote vector $[t_{1,1},\cdots,t_{K,N}]$ and $[x_{1,1},\cdots,x_{K,N}]$, respectively. $\forall \alpha,\beta$ *such that* $\alpha+\beta=1$, according to lemma 1, we have

$$\begin{aligned} h(\alpha\vec{t}_1+\beta\vec{t}_2,\alpha\vec{x}_1+\beta\vec{x}_2) &= \sum_{k=1}^{K}\sum_{n=1}^{N} C_{k,n} h_{k,n}(\alpha t_{1k,n}+\beta t_{2k,n},\alpha x_{1k,n}+\beta x_{2k,n}) \\ &\le \sum_{k=1}^{K}\sum_{n=1}^{N} C_{k,n}[\alpha h_{k,n}(t_{1k,n},x_{1k,n})+\beta h_{k,n}(t_{2k,n},x_{2k,n})] \\ &= \alpha\sum_{k=1}^{K}\sum_{n=1}^{N} C_{k,n} h_{k,n}(t_{1k,n},x_{1k,n})+\beta\sum_{k=1}^{K}\sum_{n=1}^{N} C_{k,n} h_{k,n}(t_{2k,n},x_{2k,n}) \\ &= \alpha h(\vec{t}_1,\vec{x}_1)+\beta h(\vec{t}_2,\vec{x}_2) \end{aligned} \tag{3.39}$$

Hence, we prove $h(t_{1,1},\cdots,t_{K,N},x_{1,1},\cdots,x_{K,N})$ is convex.

Actually, we can also prove Lemma 2 by calculate its eigenvalues directly. The Hessian H of function $h(t_{1,1},\cdots,t_{K,N},x_{1,1},\cdots,x_{K,N})$ is given by

$$\begin{bmatrix} \frac{\partial^2 h}{\partial x_{1,1}^2} & \frac{\partial^2 h}{\partial x_{1,1}\partial t_{1,1}} & \frac{\partial^2 h}{\partial x_{1,1}\partial x_{1,2}} & \frac{\partial^2 h}{\partial x_{1,1}\partial t_{1,2}} & \cdots & \frac{\partial^2 h}{\partial x_{1,1}\partial x_{K,N}} & \frac{\partial^2 h}{\partial x_{1,1}\partial t_{K,N}} \\ \frac{\partial^2 h}{\partial t_{1,1}\partial x_{1,1}} & \frac{\partial^2 h}{\partial t_{1,1}^2} & \frac{\partial^2 h}{\partial t_{1,1}\partial x_{1,2}} & \frac{\partial^2 h}{\partial t_{1,1}\partial t_{1,2}} & \cdots & \frac{\partial^2 h}{\partial t_{1,1}\partial x_{K,N}} & \frac{\partial^2 h}{\partial t_{1,1}\partial t_{K,N}} \\ & & & \vdots & & & \\ \frac{\partial^2 h}{\partial x_{K,N}\partial x_{1,1}} & \frac{\partial^2 h}{\partial x_{K,N}\partial t_{1,1}} & \frac{\partial^2 h}{\partial x_{K,N}\partial x_{1,2}} & \frac{\partial^2 h}{\partial x_{K,N}\partial t_{1,2}} & \cdots & \frac{\partial^2 h}{\partial x_{K,N}^2} & \frac{\partial^2 h}{\partial x_{K,N}\partial t_{K,N}} \\ \frac{\partial^2 h}{\partial t_{K,N}\partial x_{1,1}} & \frac{\partial^2 h}{\partial t_{K,N}\partial t_{1,1}} & \frac{\partial^2 h}{\partial t_{K,N}\partial x_{1,2}} & \frac{\partial^2 h}{\partial t_{K,N}\partial t_{1,2}} & \cdots & \frac{\partial^2 h}{\partial t_{K,N}\partial x_{K,N}} & \frac{\partial^2 h}{\partial t_{K,N}^2} \end{bmatrix} \quad (3.40)$$

After simplification, the Hessian H is of the form

$$\begin{bmatrix} \frac{\partial^2 h}{\partial x_{1,1}^2} & \frac{\partial^2 h}{\partial x_{1,1}\partial t_{1,1}} & 0 & 0 & \cdots & 0 & 0 \\ \frac{\partial^2 h}{\partial t_{1,1}\partial x_{1,1}} & \frac{\partial^2 h}{\partial t_{1,1}^2} & 0 & 0 & \cdots & 0 & 0 \\ & & \ddots & & & & \\ 0 & 0 & 0 & 0 & \cdots & \frac{\partial^2 h}{\partial x_{K,N}^2} & \frac{\partial^2 h}{\partial x_{K,N}\partial t_{K,N}} \\ 0 & 0 & 0 & 0 & \cdots & \frac{\partial^2 h}{\partial t_{K,N}\partial x_{K,N}} & \frac{\partial^2 h}{\partial t_{K,N}^2} \end{bmatrix} = \begin{bmatrix} H_{1,1} & 0 & 0 & 0 & \cdots & 0 \\ 0 & H_{1,2} & 0 & 0 & \cdots & 0 \\ 0 & 0 & H_{1,3} & 0 & \cdots & 0 \\ 0 & 0 & 0 & H_{1,4} & \cdots & 0 \\ & & & & \ddots & \\ 0 & 0 & 0 & 0 & & H_{K,N} \end{bmatrix} \quad (3.41)$$

where $H_{k,n} = \begin{bmatrix} \frac{\partial^2 h}{\partial x_{k,n}^2} & \frac{\partial^2 h}{\partial x_{k,n}\partial t_{k,n}} \\ \frac{\partial^2 h}{\partial t_{k,n}\partial x_{k,n}} & \frac{\partial^2 h}{\partial t_{k,n}^2} \end{bmatrix}$. Then, we calculate the eigenvalues and eigenvectors of the above matrix H.

We use λ to denote the eigenvalue of H. As we all known, eigenvalue is the root of the equation

$$\det(H - \lambda I) = 0 \quad (3.42)$$

After substituting (3.41) into (3.42), we have

$$\det\begin{bmatrix} H_{1,1}-\lambda I & 0 & 0 & 0 & \cdots & 0 \\ 0 & H_{1,2}-\lambda I & 0 & 0 & \cdots & 0 \\ 0 & 0 & H_{1,3}-\lambda I & 0 & \cdots & 0 \\ 0 & 0 & 0 & H_{1,4}-\lambda I & \cdots & 0 \\ & & & & \ddots & \\ 0 & 0 & 0 & 0 & & H_{K,N}-\lambda I \end{bmatrix}=0 \tag{3.43}$$

According to the rule of determinant expansion by minors, we can simplify determinant of H to

$$(\frac{\partial^2 h}{\partial x_{1,1}^2}-\lambda)\begin{bmatrix} \frac{\partial^2 h}{\partial t_{1,1}^2}-\lambda & 0 & 0 & 0 & \cdots & 0 & 0 \\ & & \ddots & & & & \\ 0 & & 0 & \cdots & & \frac{\partial^2 h}{\partial x_{K,N}^2}-\lambda & \frac{\partial^2 h}{\partial x_{K,N}\partial t_{K,N}} \\ 0 & & 0 & \cdots & & \frac{\partial^2 h}{\partial t_{K,N}\partial x_{K,N}} & \frac{\partial^2 h}{\partial t_{K,N}^2}-\lambda \end{bmatrix}$$

$$-\frac{\partial^2 h}{\partial x_{1,1}\partial t_{1,1}}\begin{bmatrix} \frac{\partial^2 h}{\partial t_{1,1}\partial x_{1,1}} & 0 & 0 & 0 & \cdots & 0 & 0 \\ & & \ddots & & & & \\ 0 & & 0 & \cdots & & \frac{\partial^2 h}{\partial x_{K,N}^2}-\lambda & \frac{\partial^2 h}{\partial x_{K,N}\partial t_{K,N}} \\ 0 & & 0 & \cdots & & \frac{\partial^2 h}{\partial t_{K,N}\partial x_{K,N}} & \frac{\partial^2 h}{\partial t_{K,N}^2}-\lambda \end{bmatrix}$$

$$=(\frac{\partial^2 h}{\partial x_{1,1}^2}-\lambda)(\frac{\partial^2 h}{\partial t_{1,1}^2}-\lambda)\det\begin{bmatrix} \frac{\partial^2 h}{\partial x_{1,2}^2}-\lambda & \frac{\partial^2 h}{\partial x_{1,2}\partial t_{1,1}} & 0 & 0 & \cdots & 0 & 0 \\ \frac{\partial^2 h}{\partial t_{1,2}\partial x_{1,1}} & \frac{\partial^2 h}{\partial t_{1,2}^2}-\lambda & 0 & 0 & \cdots & 0 & 0 \\ & & \ddots & & & & \\ 0 & 0 & 0 & 0 & \cdots & \frac{\partial^2 h}{\partial x_{K,N}^2}-\lambda & \frac{\partial^2 h}{\partial x_{K,N}\partial t_{K,N}} \\ 0 & 0 & 0 & 0 & \cdots & \frac{\partial^2 h}{\partial t_{K,N}\partial x_{K,N}} & \frac{\partial^2 h}{\partial t_{K,N}^2}-\lambda \end{bmatrix}$$

$$
-\frac{\partial^2 h}{\partial x_{1,1}\partial t_{1,1}}\frac{\partial^2 h}{\partial t_{1,1}\partial x_{1,1}}\det\begin{bmatrix}
\frac{\partial^2 h}{\partial x_{1,2}^2}-\lambda & \frac{\partial^2 h}{\partial x_{1,2}\partial t_{1,1}} & 0 & 0 & \cdots & 0 & 0\\
\frac{\partial^2 h}{\partial t_{1,2}\partial x_{1,1}} & \frac{\partial^2 h}{\partial t_{1,2}^2}-\lambda & 0 & 0 & \cdots & 0 & 0\\
 & & & \ddots & & & \\
0 & 0 & 0 & 0 & \cdots & \frac{\partial^2 h}{\partial x_{K,N}^2}-\lambda & \frac{\partial^2 h}{\partial x_{K,N}\partial t_{K,N}}\\
0 & 0 & 0 & 0 & \cdots & \frac{\partial^2 h}{\partial t_{K,N}\partial x_{K,N}} & \frac{\partial^2 h}{\partial t_{K,N}^2}-\lambda
\end{bmatrix}
$$

$$
=\begin{pmatrix}
(\frac{\partial^2 h}{\partial x_{1,1}^2}-\lambda)(\frac{\partial^2 h}{\partial t_{1,1}^2}-\lambda)-\\
\frac{\partial^2 h}{\partial x_{1,1}\partial t_{1,1}}\frac{\partial^2 h}{\partial t_{1,1}\partial x_{1,1}}
\end{pmatrix}\det\begin{bmatrix}
\frac{\partial^2 h}{\partial x_{1,2}^2}-\lambda & \frac{\partial^2 h}{\partial x_{1,2}\partial t_{1,1}} & 0 & 0 & \cdots & 0 & 0\\
\frac{\partial^2 h}{\partial t_{1,2}\partial x_{1,1}} & \frac{\partial^2 h}{\partial t_{1,2}^2}-\lambda & 0 & 0 & \cdots & 0 & 0\\
 & & & \ddots & & & \\
0 & 0 & 0 & 0 & \cdots & \frac{\partial^2 h}{\partial x_{K,N}^2}-\lambda & \frac{\partial^2 h}{\partial x_{K,N}\partial t_{K,N}}\\
0 & 0 & 0 & 0 & \cdots & \frac{\partial^2 h}{\partial t_{K,N}\partial x_{K,N}} & \frac{\partial^2 h}{\partial t_{K,N}^2}-\lambda
\end{bmatrix}
$$

$$
=\begin{pmatrix}
(\frac{\partial^2 h}{\partial x_{1,1}^2}-\lambda)(\frac{\partial^2 h}{\partial t_{1,1}^2}-\lambda)-\\
\frac{\partial^2 h}{\partial x_{1,1}\partial t_{1,1}}\frac{\partial^2 h}{\partial t_{1,1}\partial x_{1,1}}
\end{pmatrix}\det\begin{bmatrix}
H_{1,2}-\lambda I & 0 & 0 & 0 & \cdots & 0\\
0 & H_{1,3}-\lambda I & 0 & 0 & \cdots & 0\\
0 & 0 & H_{1,4}-\lambda I & 0 & \cdots & 0\\
0 & 0 & 0 & H_{1,5}-\lambda I & \cdots & 0\\
 & & & & \ddots & \\
0 & 0 & 0 & 0 & & H_{K,N}-\lambda I
\end{bmatrix}
$$

$$
=\det(H_{1,1}-\lambda I)\det\begin{bmatrix}
H_{1,2}-\lambda I & 0 & 0 & 0 & \cdots & 0\\
0 & H_{1,3}-\lambda I & 0 & 0 & \cdots & 0\\
0 & 0 & H_{1,4}-\lambda I & 0 & \cdots & 0\\
0 & 0 & 0 & H_{1,5}-\lambda I & \cdots & 0\\
 & & & & \ddots & \\
0 & 0 & 0 & 0 & & H_{K,N}-\lambda I
\end{bmatrix}. \tag{3.44}
$$

Use the above formula on Hessian of H again and again, finally, we can simplify (3.43) to

$$\det(H-\lambda I)=\det(H_{1,1}-\lambda I)\det(H_{1,2}-\lambda I)\det(H_{1,3}-\lambda I)\cdots\det(H_{K,N}-\lambda I)=0 \quad (3.45)$$

Therefore, we can conclude the set of eiginvalues of H is the union of all the eigenvalues of $H_{k,n}$ for all k and n. According to Lemma 1, we know the two eigenvalues of $H_{k,n}$ are nonnegative. Hence, the Hessian H of $h(t_{1,1},\cdots,t_{K,N},x_{1,1},\cdots,x_{K,N})$ is positive semidefinite. Now we prove *Second order conditions* of convex are satisfied for function $h(t_{1,1},\cdots,t_{K,N},x_{1,1},\cdots,x_{K,N})$. Hence, Lemma 2 is proved.

Lemma 3

Given $h(x)=c_1h_1(x)+c_2h_2(x)$, $x\in R^n, c_1>0, c_2>0$. if $h_1(x)$ and $h_2(x)$ are convex functions, $h(x)$ is also a convex function.

Proof:

$\forall \alpha,\beta$ *such that* $\alpha+\beta=1$, we have

$$\begin{aligned} h(\alpha x+\beta y) &= c_1h_1(\alpha x+\beta y)+c_2h_2(\alpha x+\beta y)\\ &\le c_1\alpha h_1(x)+c_1\beta h_1(y)+c_2\alpha h_2(x)+c_2\beta h_2(y)\\ &= \alpha[c_1h_1(x)+c_2h_2(x)]+\beta[c_1h_1(x)+c_2h_2(y)]\\ &= \alpha h(x)+\beta h(x) \end{aligned} \quad (3.46)$$

Theorem 1

$f(t,x)=\sum_{m=1}^{M}\frac{1}{m!}\sum_{k=1}^{K}\sum_{n=1}^{N}C_{k,n}t_{k,n}^{1-m}x^{m}{}_{k,n}$ is a convex function.

Proof:

We define $h_m(t_{1,1},\cdots,t_{K,N},x_{1,1},\cdots,x_{K,N})=\sum_{k=1}^{K}\sum_{n=1}^{N}C_{k,n}t_{k,n}^{1-m}x^{m}{}_{k,n}$. According to lemma 2, we can easily know that h_m is a convex function. We can express $f(t,x)$ as $f(t,x)=\sum_{m=1}^{M}\frac{1}{m!}h_m(t,x)$. According to lemma 3, the summation of convex function is also a convex function. Hence, theorem 1 is proved.

Up to now, we have shown that all the three requirements are satisfied. Therefore, we successfully transform our problem to a standard convex programming problem on a certain convex set, which is given by $t_{k,n} \geq 0$ and $x_{k,n} \geq 0$ for all k and n.

3.3.2 Solve the Convex Programming Problem

Convex nonlinear programming has been a hot research topic for many years. Up to now, many efficient algorithms to solve this kind of problem have already been proposed by many researchers. At the same time, many software programs based on these algorithms are developed by research groups in universities and software companies. Currently, the popular solvers are MOSEK, MINOS, KNITRO, which are commercial software, and SOLNP, which is a free solver.

Among those algorithms, interior point method has been proved to be effective and efficient for convex programming. Therefore, by adopting this method, we have a near optimal approach for multiple users to TCDMA a multicarrier communication system. Because we only have finite terms in the Taylor expansion to approximate the original objective function, the approach is called 'near optimal', but not 'optimal'. Before we investigate the performance of our approach in Section 3.6, we briefly introduce the interior point method.

3.3.2.1 Interior Point Method

Interior point methods in mathematical programming have been the largest and most dramatic area of research in optimization since the development of the simplex method for linear programming. Over the last thirteen years, interior point methods have attracted some of the very best researchers in operations research, applied mathematics, and computer science. Approximately 2, 000 papers have been written on the subject following the seminal work of Karmarkar [9]. Interior point methods have permanently changed the landscape of mathematical programming theory, practice, and computation. Linear programming is no longer synonymous with the celebrated simplex method, and many researchers now tend to view linear programming more as a special case of nonlinear programming due to these developments.

Almost immediately after Karmarmkar's work appeared, researchers began to explore extensions of interior point methods to general convex optimization problems. Indeed, the nonlinear nature of interior point methods naturally suggested that such extensions were possible. Throughout the 1980's, a number of papers were written that showed that central path methods and potential reduction methods for linear programming problems could be generalized to certain types of convex programs with theoretical performance guarantees, under a variety of restrictions (such as smoothness conditions) on the convex functions involved. However, there was no unifying theory or analysis. Then, Nesterov and Nemirovskii [10] presented a deep and unified theory of interior point methods for all of convex programming based on the notion of *self-concordant functions*. Details about interior point method for convex programming can also be found in [11].

3.3.2.2 Interior Point Method for Linear Programming

To begin our description of interior point methods for linear programming, we consider the linear programming problem in standard form:

$$P: minimize \quad c^T x \tag{3.47}$$

$$\text{subject to} \quad Ax = b \tag{3.48}$$

$$x \geq 0 \tag{3.49}$$

where x is a vector of n variables, whose standard linear programming dual problem is:

$$D: maximize \quad b^T y \tag{3.50}$$

$$\text{subject to} \quad A^T y + s = c \tag{3.51}$$

$$s \geq 0 \tag{3.52}$$

where s are slack variables. Given a feasible solution x of Problem P and a feasible solution (y,s) of D, the duality gap is simply $c^T x - b^T y = x^T s \geq 0$.

We introduce the following notation which will be very convenient for manipulating equations, etc. A feasible solution x of P is *strictly feasible* if $x > 0$, and a feasible solution (y,s) of D is *strictly feasible* if $s > 0$. Let e denote the vector of ones, i.e., $e = (1, \cdots, 1)^T$. Suppose that $x > 0$. Define the matrix X to be the $n \times n$ diagonal matrix whose diagonal entries are precisely the components of x. Then X looks like:

$$\begin{pmatrix} x_1 & 0 & \cdots & 0 \\ 0 & x_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & x_n \end{pmatrix}.$$

Notice that $Xe = x$, and $X^{-1}e = (\frac{1}{x_1}, \cdots, \frac{1}{x_n})^T$. Also, notice that both X and X^{-1} are positive-definite symmetric matrices.

There are many different types of interior point algorithms for linear programming, with certain common mathematical themes having to do with the logarithmic barrier function. In my opinion, most interior point algorithms fall into one of three main categories: affine scaling methods, potential reduction methods, and central path methods. Among them, central path methods are by far the most useful in theory and the most used in practice. Thus, we now briefly summarize this kind of algorithm.

The *central path* of the linear program problem P is obtained as the solution to an amended version of P, where a parameterized logarithmic barrier term is added to the objective function. Consider the logarithmic barrier problem $BP(\mu)$ parameterized by the positive barrier parameter μ:

$$BP(\mu): \quad minimize \quad c^T x - \mu \sum_{j=1}^{n} \ln(x_j) \tag{3.53}$$

$$\text{subject to} \quad Ax = b \tag{3.54}$$

$$x > 0 \tag{3.55}$$

The Karush-Kuhn-Tucker conditions for $BP(\mu)$ are:

$$Ax = b, x > 0 \tag{3.56}$$

$$c - \mu X^{-1} e = A^T y \tag{3.57}$$

If we define $s = \mu X^{-1} e$, then we can rewrite these optimality conditions as:

$$Ax = b, x > 0 \tag{3.58}$$

$$A^T y + s = c, s > 0 \tag{3.59}$$

$$XSe - \mu e = 0 \tag{3.60}$$

Let $(x(\mu), y(\mu), s(\mu))$ denote the solution to system (3.58-3.40) for the given positive parameter μ. Then the set $\mathrm{T} = \{(x(\mu), y(\mu), s(\mu)) \mid \mu > 0\}$ is defined to be the *central path* of the linear program problem P. As μ is reduced to zero, the central path will converges to the optimal solution of the linear programming problem. From the first two equation systems of (3.58-3.40), we see that a solution $(x(\mu), y(\mu), s(\mu))$ along the central path is strictly feasible for the primal and the dual problem, and that the duality gap on the central path is $x^T s = e^T XSe = \mu e^T e = \mu n$, which follows from equation (3.40). Substituting this equation into (3.40), we obtain the following equivalent and parameter-free characterization of the central path:

$$Ax = b, x > 0 \tag{3.61}$$

$$A^T y + s = c, s > 0 \tag{3.62}$$

$$XSe - (x^T s / n)e = 0 \tag{3.63}$$

Equation (3.63) is precisely where the nonlinearity arises, and in general it is not possible to solve (3.61-3.63) in closed form except in trivial cases.

The strategy in most central path methods is to solve for approximate solutions along the central path for a decreasing sequence of the duality gap (or equivalently, of the barrier parameter μ) that tends to zero in the limit. There are a number of ways to carry out this strategy. For example, for a given value of the duality gap or of the barrier parameter μ, one can choose to approximately optimize $BP(\mu)$ or, equivalently, to approximately solve (3.61-3.63), or to approximately solve some other equivalent characterization of the central path. Also, one can choose a number of ways to approximately solve the system of nonlinear equations under consideration (Newton's method is one obvious choice, as are predictor-corrector methods and other higher-order methods, preconditioned conjugate gradient methods, etc.). Overlayed with all of this is the way in which the numerical linear algebra is implemented. Furthermore, one needs to decide how to measure "approximate" in the approximate solution. Last of all, there is considerable leeway in developing a strategy for reducing the duality gap (or the barrier parameter μ) at each iteration. (For example, aggressively shrinking the duality gap seems like a good idea, but will also increase the number of iterations of Newton's method (or other method) that is used to re-solve (approximately) the new system of nonlinear equations.)

3.3.2.3 An Interior Point Framework for Our Problem

The extension of interior point method from linear programming to convex programming is straightforward. Before we propose our algorithm, we simplify the form of our problem by introduce some slack variables. The constrain (3.35) $\sum_{k=1}^{K} t_{k,n} \le 1$ can be reformed as $s_n + \sum_{k=1}^{K} t_{k,n} = 1, s_n \ge 0$ with slack variable s_n. Then, we eliminate the inequality constraints and our problem becomes a convex programming with linear equality constraints in the following form:

$$\text{Minimize } f(x) \tag{3.64}$$

subject to

$$Ax = b \tag{3.65}$$

$$x \geq 0 \tag{3.66}$$

Very likely as the optimality conditions system (3.61-3.63) of the linear programming, we know the optimal solution of the above problem is located by solve the following nonlinear equations:

$$Ax = b, x > 0 \tag{3.67}$$

$$\nabla f(x) + A^T \omega - z = 0, z > 0 \tag{3.68}$$

$$XZe - \mu e = 0 \tag{3.69}$$

where X and Z are diagonal matrix whose diagonal elements are corresponding elements in vector x and z.

We use the classical Newton's method to solve the above nonlinear equations. Newton's method is an iterative approach designed to solve the following set of r equations:

$$f_i(y) = 0, i = 1, \cdots r \quad \text{or} \quad \mathbf{f}(y) = 0 \tag{3.70}$$

At the k th iteration, by solve the equations of

$$\mathbf{f}(y^k) + \mathbf{J}(y^k)\mathbf{d} = 0 \tag{3.71}$$

where $\mathbf{J}(y^k)$ is the Jacobian of $\mathbf{f}(y)$ at y^k, we obtain the change of position for the solution y by taking $y^{k+1} = y^k + \mathbf{d}$. Solving (3.71) for $\mathbf{d}$ leads to

$$\mathbf{d} = -\mathbf{J}(y^k)^{-1}\mathbf{f}(y^k) \tag{3.72}$$

If we start at an initial point y^0 and update y as $y^{k+1} = y^k + \mathbf{d}$, after sufficient number of iterations, y^k will converge to the solution of (3.70)

The Jacobian of (3.67-3.69) is:

$$\begin{pmatrix} A & 0 & 0 \\ H & A^T & -I \\ Z & 0 & X \end{pmatrix} \tag{3.73}$$

If we define the Newton direction $\mathbf{d} \triangleq [\Delta x, \Delta\omega, \Delta z]^T$, it can be obtained by solving

$$\begin{pmatrix} A & 0 & 0 \\ H & A^T & -I \\ Z^k & 0 & X^k \end{pmatrix}\begin{pmatrix} \Delta x \\ \Delta\omega \\ \Delta z \end{pmatrix} = \begin{pmatrix} b - Ax^k \\ -\nabla f(x^k) - A^T\omega^k + z^k \\ \mu - X^k z^k \end{pmatrix} \tag{3.74}$$

where H is the Hessian of $f(x)$ at x^k.

After we got the direction $\mathbf{d}$, we move the current point (x^k, ω^k, z^k) to a new point $(x^{k+1}, \omega^{k+1}, z^{k+1})$ by taking a step in direction $\mathbf{d}$. Because we must guarantee $x > 0$ and $z > 0$, at each iteration to update x, ω, and z, we must determine the step size carefully to avoid violate this constraints. The step sizes, α_P and α_D, are chosen in two spaces to preserve $x > 0$ and $z > 0$. This requires a ratio test to obtain α_P and α_D as follows

$$\alpha_P = \gamma \min_j \left\{ \frac{-x_j}{\Delta x_j} : \Delta x_j < 0 \right\} \tag{3.75}$$

$$\alpha_D = \gamma \min_j \left\{ \frac{-z_j}{\Delta z_j} : \Delta z_j < 0 \right\} \tag{3.76}$$

where γ is the step size factor that keeps us from actually touching the boundary. Typically, $\gamma = 0.995$. Finally, we know the new point is

$$x^{k+1} = x^k + \alpha_P \Delta x \tag{3.77}$$

$$\omega^{k+1} = \omega^k + \alpha_D \Delta\omega \tag{3.78}$$

$$z^{k+1} = z^k + \alpha_D \Delta z \tag{3.79}$$

which completes one iteration. Furthermore, we reduce μ in every iteration to make (x^k, ω^k, z^k) converges to optimal solution faster.

We then summarize the interior point algorithm to solve our problem as follows:

1. (Initialization) Start with some feasible point $x^0 > 0, \omega^0, z^0 > 0$ and set iteration counter $k = 0$
2. (Optimality test) if dual gap is smaller than the optimality tolerance ε, that is $(x^k)^T z^k < \varepsilon$, stop; otherwise, go to step 3.
3. Obtain Newton direction $\mathbf{d}$.
4. Find step sizes α_P and α_D.
5. Update solution from (x^k, ω^k, z^k) to $(x^{k+1}, \omega^{k+1}, z^{k+1})$ and reduce μ.
6. Put $k = k+1$ and go to step 2.

With the above algorithm, we can solve our convex problem (3.33-3.37). Therefore, up to now, by adopting this method, we have a near optimal approach for multiple users to TCDMA a multicarrier communication system. Why this approach can not achieve optimality of our original problem perfectly is because we only have finite terms in the Taylor expansion to approximate the original objective function. Comparing original objective function in (3.28) and the reformulated objective (3.33), we observe that only totally first M are used to approximate the original objective function. Thus, although the reformulated function will be very close to the original one if we chose sufficiently large M, strictly speaking, our approach based on the interior point method is a near optimal but not the optimal one. However, we can fill up this gap by increase the value of M. The performance of this near optimal approach is investigated in Section 3.6.

3.4 Ad Hoc CTDMA Approach

To reduce computational complexity, in this section, we propose a simplified ad hoc approach for users to share all code channels. We consider a multicarrier system with K codes and N users, where $K \leq N$. We assume one code can only be assigned to one user in a whole time slot. In other words, in a time slot, a user will exclusively occupy a code channel. Therefore, we restrict $t_{k,n}$ to be binary variables, i.e., $t_{k,n}$ can only take values of 0 or 1.

3.4.1 Fully Loaded System

3.4.1.1 Problem Formulation

Consider a multicarrier system with N codes. If there are totally N simultaneous users in the system, we call the system a fully loaded system. We then consider the ad hoc approach for the fully loaded system. Because $t_{k,n} = 0$ or 1 for all k and n, according to (3.13), we sum over all n and finally have

$$\sum_{n=1}^{N}\sum_{k=1}^{N} t_{k,n} \leq N \tag{3.80}$$

Then we conclude

$$\sum_{k=1}^{N} t_{k,n} = 1 \quad \text{for all} \quad k \tag{3.81}$$

We prove (3.81) as follows:

As $t_{k,n}$ is binary variable, suppose $\sum_{n=1}^{N} t_{k,n} \neq 1$ for a particular k^*, we have two cases $\sum_{n=1}^{N} t_{k^*,n} = 0$ or $\sum_{n=1}^{N} t_{k^*,n} \geq 2$. The former case is obviously impossible because $\sum_{n=1}^{N} r_{k^*,n} t_{k^*,n} = R_{k^*}$ and $t_{k^*,n} \geq 0$ for all n. For the latter case, if $\sum_{n=1}^{N} t_{k^*,n} \geq 2$, because of $\sum_{n=1}^{N}\sum_{k=1}^{N} t_{k,n} \le N$, we have

$$\sum_{n=1}^{N}\sum_{k=1,k\neq k^{*}}^{N} t_{k,n} \le N-2 \tag{3.82}$$

This means there must exist an index $k^{'}$, such that $\sum_{n=1}^{N} t_{k^{'},n} = 0$. This contradicts

the constrain of $\sum_{n=1}^{N} r_{k^{'},n} t_{k^{'},n} = R_{k^{'}}$. Therefore, we conclude (3.81) holds.

Because $t_{k,n}$ is binary variable, (3.81) actually imply that one user can only be assigned only one code, i.e., there only exist one $t_{k,n}$ to be 1 for all k, while others are all zero. With (3.81), we can simplify constraint (3.12) to a simple case. It is evidently $r_{k,n}$ must satisfy the following equation:

$$r_{k,n} = \begin{cases} R_k & when \quad t_{k,n} = 1 \\ any\ value & when \quad t_{k,n} = 0 \end{cases} \quad \text{for all } k. \tag{3.83}$$

Taking 'any value' in (3.83) as R_k and substituting equation (3.83) to (3.12), we have will have a simplified constrain

$$\sum_{n=1}^{N} t_{k,n} = 1, \text{ for all } k \tag{3.84}$$

which is exactly as same as (3.81). With the above assumption and simplification, our original problem becomes an integer linear programming problem, given by

$$\text{Minimize } P = \sum_{k=1}^{K}\sum_{n=1}^{N} t_{k,n}(2^{R_k/B}-1)C_{k,n} \tag{3.85}$$

subject to

$$\sum_{n=1}^{N} t_{k,n} = 1 \text{ for all } k \tag{3.86}$$

$$\sum_{k=1}^{N} t_{k,n} \leq 1 \text{ for all } n \tag{3.87}$$

$$t_{k,n} = 0 \text{ or } 1 \text{ for all } k \text{ and } n \tag{3.88}$$

where $C_{k,n} = \dfrac{\sigma^2 \mathbf{C}_n^H \mathbf{A}_k^{-1} \mathbf{A}_k^{-1H} \mathbf{C}_n}{g_k^2}$.

3.4.1.2 One-to-one Assignment

We then further explain the insight of our simplification in previous section. We assume the system is a system with N users and N codes. Evidently, different user in the system should have different code since each user's data should be separate from others'. Hence, one code can be only assigned to one user. That is exactly the meaning of constraint (3.87). Furthermore, because we assume one user will occupy a code channel exclusively in a time slot, it means one code can only be assigned to one user. Then, evidently, each user should be assigned only one code because there are totally only N codes and one code only serve one user. If one user has multiple codes, there must be some users who will have no code to be assigned with. This is exact the meaning of equation (3.86). So, we call this ad hoc CTDMA approach ***one-to-one assignment*** approach because one code can only be assigned to one user and one user can only have one code.

In order to obtain the one-to-one code assignment, i.e., to solve the problem (3.85-3.88), we can have two methods. One is based on linear programming and the other is based on Hungarian method. Actually, the two methods give the same optimal code assignment solution and we will discuss them one by one.

3.4.1.2.1 Solve the Problem by Linear Programming

By the integer solution property [12], the last condition (3.88) can be replaced by

$t_{k,n} \geq 0$ and the problem becomes a linear programming problem of the form

$$\text{Minimize } P = \sum_{k=1}^{K} \sum_{n=1}^{N} t_{k,n} p_{k,n} \tag{3.89}$$

subject to

$$\sum_{n=1}^{N} t_{k,n} = 1 \text{ for all } k \tag{3.90}$$

$$\sum_{k=1}^{N} t_{k,n} \leq 1 \text{ for all } n \tag{3.91}$$

$$t_{k,n} \geq 0 \text{ for all } k \text{ and } n \tag{3.92}$$

where $p_{k,n} = \dfrac{(2^{R_k/B}-1)\sigma^2 \mathbf{C}_n^H \mathbf{A}_k^{-1} \mathbf{A}_k^{-1H} \mathbf{C}_n}{g_k^2}$.

The development of linear programming has been ranked among the most important scientific advances of the mid-20th century, and we must agree with the assessment. Its impact since just 1950 has been extraordinary. Today, it is a standard tool to solve problems in many scientific fields.

A standard linear programming problem can be described by the following form:

$$\text{Minimize } c^T x \tag{3.93}$$

$$\text{subject to } Ax = B \tag{3.94}$$

$$x \geq 0 \tag{3.95}$$

where c^T and B are a constant vector, i.e., each element of is a constant. A is a constant matrix whose each entry is also a constant.

Other forms of linear programming can be converted to the standard form easily. Maximization of $c^T x$ is equivalent to the minimization of $(-c)^T x$. Inequality constraint can be converted to equality constraint by introducing *slack variables*, i.e., $\sum_{i=1}^{N} a_i x_i \le b_i$ is equivalent to $\varsigma + \sum_{i=1}^{N} a_i x_i = b_i$ and $\varsigma \ge 0$. Furthermore, we notice that $\sum_{i=1}^{N} a_i x_i \ge b_i$ is equivalent to $\sum_{i=1}^{N} a_i x_i - \varsigma = b_i$ and $\varsigma \ge 0$. Therefore, we only need to focus on the standard form of linear optimization.

Linear optimization problem has been a hot topic in operation research for many years. In 1947, George Dantzig developed the *simplex method*, a remarkable efficient method that is used routinely to solve linear programming problem. The idea of the simplex method is quite simple. We start off from a vertex, which is also called a *basic feasible solution*, then attempt to move along an edge of the feasible region, i.e., the region of x that satisfy the constraints, to another vertex toward the direction of optimization. We make sure that each move does not increase the objective function. As this thesis is not on the topic of operation research, we omit the details about this common algorithm. A very good introduction of simplex method and linear programming can be found in [13]. Based on the simplex method, a class of linear programming package, such as LP_Solve, CPLEX, GULF, is developed by researchers and companies. These software packages can solve even very large-scale linear programming problem with high efficiency.

The announcement by Karmarkar [14] in 1984 that he had developed a fast algorithm that generated iterates that lie in the interior of the feasible set (rather than on the boundary, as simplex methods do) opened up exciting new avenues for research in both the computational complexity and mathematical programming communities. Since then, there has been intense research into a variety of methods that maintain strict feasibility of all iterates, at least with respect to the inequality constraints. Although dwarfed in volume by simplex-based packages, interior-point products such as CPLEX/Barrier, LOQP, and OSL have emerged and have proven to be competitive with, and often superior to, the best simplex packages, especially on large problems.

3.4.1.2.2 Solve the Problem by Hungarian Method

Assignment problems deal with the questions how to assign N items (e.g. jobs) to N machines (or workers) in the best possible way, i.e., the way with the minimum total cost where the cost of assigning job n to machine k is $C_{k,n}$. All the costs are assumed to be nonnegative. They consist of three components: the cost matrix $\mathbf{C}$ with (k,n) th entry of $C_{k,n}$, the assignment as underlying combinatorial structure, and an objective function modeling the best way.

Consider a system with N users and N codes. As we have mentioned in Section 3.4.1.2, in a time slot, one code can only be assigned to one user and one user can only be assigned with one code. The power required when the k th user is assigned the n th code is given by

$$p_{k,n} = t_{k,n}(2^{R_k/B} - 1)C_{k,n} \tag{3.96}$$

We can, therefore, obtain a N by N cost matrix $\mathbf{P}$ where the (k,n) th entry $p_{k,n}$ is the power required when the k th user is assigned the n th code. If we view the codes as jobs and users as machines, with this cost matrix $\mathbf{P}$, our problem reduces to the standard assignment problem, which can be solved by the Hungarian method (or linear programming).

The Hungarian algorithm is an algorithm for solving a matching problem or more generally an assignment linear programming problem. The name was given by H. W. Kuhn in recognition of the work of the two mathematicians J.Egerváry and D.König. The Hungarian Algorithm is actually a special case of the primal-dual Algorithm. It takes a bipartite graph and produces a maximal matching. This algorithm published in 1955 can reduce the complexity of solving assignment problems from $O(M!)$ to $O(M^3)$. Here, we only outline the elaborate algorithm while the details can be found in [15] or other publications on operation research.

Outline of the Hungarian Method: Let $\mathbf{C}$ be the $N \times N$ cost matrix, whose (k,n) th element is $C_{k,n}$.

1. Minimum entries are found for each separate row of $\mathbf{C}$ and subtracted from each entry in the respective rows. The original matrix $\mathbf{C}$ is replaced by the new matrix.
2. Minimum entries are found for each separate column of A and these are subtracted from each entry in the respective columns. The original matrix (from 1) is replaced by the new matrix.
3. Cross out the minimal number of rows and columns in the matrix $\mathbf{C}$ to cover all 0 entries of $\mathbf{C}$.
4. (Termination) If there are N lines, then an optimal solution can be read off the matrix
5. If less than N lines are used to cover, go to 6.
6. (Iteration) Find the minimal entry not crossed out in 3. Add this entry to all elements that are doubly crossed out (by both a horizontal and vertical line) in 3 and subtract from all entries of $\mathbf{C}$ that are not crossed out. Return to step 3 with the new matrix.

3.4.2 Non-fully Loaded System

3.4.2.1 Problem Formulation

Consider the case where there are K users and $K < N$, where N is the total number of codes in the multicarrier communication system, which is call a non-fully loaded System. Because $t_{k,n} = 0$ or 1 for all k and n, if we define $R_{k,n} \triangleq r_{k,n} t_{k,n}$, we have $t_{k,n} \geq sign(R_{k,n})$ where $sign(x) = \begin{cases} -1 & if \quad x<0 \\ 0 & if \quad x=0 \\ 1 & if \quad x>0 \end{cases}$. Thus, the constrain (3.13) $\sum_{k=1}^{K} t_{k,n} \leq 1$ can be replace by $\sum_{k=1}^{K} sign(R_{k,n}) \leq \sum_{k=1}^{K} t_{k,n} \leq 1$ for all n. That is, in the vector $[R_{1,n}, R_{2,n}, \cdots, R_{K,n}]$, there only exists one positive element $R_{k',n}$ while others are zero. Actually, this transformed constrain implies that the nth code can be assigned to serve one user (the k'th user) with a data rate of $r_{k',n}$. This is exactly the same physical insight of constraint (3.13).

It is easy to check the following equality holds,

$$t_{k,n}(2^{r_{k,n}/B}-1)C_{k,n} = C_{k,n}(2^{R_{k,n}/B}-1) \tag{3.97}$$

We prove (3.97) as follows:

When $t_{k,n}=0$, $R_{k,n}=r_{k,n}t_{k,n}=0$, the equality holds evidently. When $t_{k,n}=1$, $R_{k,n}=r_{k,n}t_{k,n}=r_{k,n}$. After substituting it into (3.97), the left-hand-side of (3.97) will convert to

$t_{k,n}(2^{r_{k,n}/B}-1)C_{k,n} = (2^{r_{k,n}/B}-1)C_{k,n} = (2^{r_{k,n}t_{k,n}/B}-1)C_{k,n} = (2^{R_{k,n}/B}-1)C_{k,n}$. Hence, we prove equation (3.97).

Hence, we can simplify our original problem (3.11-3.15) to

$$\text{Minimize} \sum_{k=1}^{K}\sum_{n=1}^{N} C_{k,n}(2^{R_{k,n}/B}-1) \tag{3.98}$$

subject to

$$\sum_{n=1}^{N} R_{k,n} = R_k \quad \text{for} \quad k=1,\ldots,K, \tag{3.99}$$

$$\sum_{k=1}^{K} sign(R_{k,n}) \leq 1 \quad \text{for} \quad n=1,\ldots,N, \tag{3.100}$$

$$R_{k,n} \geq 0 \quad \text{for all } k \text{ and } n \tag{3.101}$$

where $sign(x) = \begin{cases} -1 & if \quad x<0 \\ 0 & if \quad x=0 \\ 1 & if \quad x>0 \end{cases}$ and $C_{k,n} = \dfrac{\sigma^2 \mathbf{C}_n^H \mathbf{A}_k^{-1} \mathbf{A}_k^{-1H} \mathbf{C}_n}{g_k^2}$.

The idea behind the formulation is straightforward. In this case, we have more codes than users in the system. Very straightforward, we can assign multiple codes to some users. The vector of $[R_{k,1}, R_{k,2}, \cdots, R_{k,N}]$ tell us the required data rate of the k th user R_k can be split into at most N nonnegative data rate $R_{k,n}$ and each positive rate can be served by one code. Because we assume one user will exclusively occupy the code assigned to him in the whole time slot, we know that one code can only be assigned to one user. That is exactly the meaning of constraint (3.100). (3.100) implies that, for each particular n, there is only one positive $R_{k,n}$, which is exactly the user to whom the code assigned.

3.4.2.2 Solve the Problem with Augmented lagrangian Penalty Method

We introduce the augmented lagrangian method to solve the above problem. We form the Lagrangian function L with a penalty term in this form

$$\begin{aligned} L = & \sum_{k=1}^{K}\sum_{n=1}^{N} \frac{(2^{R_{k,n}/B}-1)\sigma^2 \mathbf{C}_n^H \mathbf{A}_k^{-1} \mathbf{A}_k^{-1H} \mathbf{C}_n}{g_k^{\ 2}} + \sum_{k=1}^{K} \lambda_k (\rho_k - \sum_{n=1}^{N} R_{k,n}) \\ & - \sum_{k=1}^{K}\sum_{n=1}^{N} \mu_{k,n} R_{k,n} + \sum_{n=1}^{N} v_n (\sum_{k=1}^{K} sign(R_{k,n}) - 1) + c(\sum_{k=1}^{K} (\rho_k - \sum_{n=1}^{N} R_{k,n})^2 \\ & + \sum_{k=1}^{K}\sum_{n=1}^{N} R_{k,n}^2 + \sum_{n=1}^{N} (\sum_{k=1}^{K} sign(R_{k,n}) - 1)^2) \end{aligned} \tag{3.102}$$

where λ_k, $\mu_{k,n}$ and v_n are the lagrange multiplier, c is the penalty parameter.

The lagrangian function L is considered as a function of $R_{k,n}$, λ_k, $\mu_{k,n}$ and v_n for

$k=1,...,K$, and for $n=1,...,N$. The derivation of with respect to $R_{k,n}$ is given by

$$\frac{\partial L}{\partial R_{k,n}}=\frac{In(2)2^{R_{k,n}/B}\sigma^2\mathbf{C}_n^H\mathbf{A}_k^{-1}\mathbf{A}_k^{-1H}\mathbf{C}_n}{Bg_k^2}-\lambda_k-\mu_{k,n}+v_n\frac{dsign(R_{k,n})}{dR_{k,n}}$$
$$+c\{2(\sum_{n=1}^{N}R_{k,n}-\rho_k)+2R_{k,n}+2(\sum_{k=1}^{K}sign(R_{k,n})-1)\frac{dsign(R_{k,n})}{dR_{k,n}}\} \quad (3.103)$$

Then, we consider an iterative approach to seek a stationary point of the Largangian function

At each step, $R_{k,n}$, λ_k, $\mu_{k,n}$, v_n, c are updated according to the following relationships:

$$R_{k,n}\leftarrow\begin{cases}R_{k,n}-\alpha_1\dfrac{\partial L}{\partial R_{k,n}} & if \quad R_{k,n}-\alpha_1\dfrac{\partial L}{\partial R_{k,n}}\geq 0\\ R_{k,n} & if \quad R_{k,n}-\alpha_1\dfrac{\partial L}{\partial R_{k,n}}<0\end{cases} \quad (3.104)$$

$$\lambda_k\leftarrow\lambda_k-\alpha_2(\rho_k-\sum_{n=1}^{N}R_{k,n}) \quad (3.105)$$

$$\mu_{k,n}\leftarrow\mu_{k,n}-\alpha_3(-R_{k,n}) \quad (3.106)$$

$$v_n\leftarrow v_n-\alpha_4(\sum_{k=1}^{K}sign(R_{k,n})-1) \quad (3.107)$$

$$c\leftarrow c+\alpha_5(\sum_{k=1}^{K}(\rho_k-\sum_{n=1}^{N}R_{k,n})^2+\sum_{k=1}^{K}\sum_{n=1}^{N}R_{k,n}^2+\sum_{n=1}^{N}(\sum_{k=1}^{K}sign(R_{k,n})-1)^2) \quad (3.108)$$

Generally speaking, we use a gradient descent algorithm to update $R_{k,n}$, λ_k, $\mu_{k,n}$, v_n while we use a gradient ascent algorithm to update c, the penalty parameter. With this method, a stationary solution can be obtained.

3.4.2.3 Solve the Problem with multiple-to-one approach

Unfortunately, it may take a long time for the above iterative approach to converge to the stationary point if the parameters, $\alpha_1,\alpha_2,\cdots,\alpha_5$, is not appropriately chosen.

Therefore, we develop a simple and heuristic method for the code assignment in non-fully loaded systems.

Evidently, when $K < N$, we have extra codes so that some user can be assigned more than one codes to lower the total transmission power. This is why we call the method multiple-to-one approach. We can consider the following algorithm.

Algorithm 1:

1. Initialization: Optimally assigned only one code to each user.
2. Compute the required power of each user.
3. Allow the user with the maximum transmission power pick one more code.
4. Optimally split data rate requirement among the multiple code of the user with maximum power.
5. Go back to 2 until all the code have been assigned.

We need further describe details of the step 1 and 4 of the above algorithm. In the initialization, we stipulate that each user can only pick one code. The optimality in the step can be achieved by solving the following problem with linear programming techenique.

$$\text{Minimize } \; P = \sum_{k=1}^{K}\sum_{n=1}^{N} t_{k,n} p_{k,n} \tag{3.109}$$

subject to

$$\sum_{n=1}^{N} t_{k,n} = 1 \; \text{ for all } \; k \tag{3.110}$$

$$\sum_{k=1}^{K} t_{k,n} \le 1 \; \text{ for all } \; n \tag{3.111}$$

$$t_{k,n} = 0 \text{ or } 1 \; \text{ for all } \; k \; \text{ and } \; n \tag{3.112}$$

where $p_{k,n} = \dfrac{(2^{R_k/B}-1)\sigma^2 \mathbf{C}_n^H \mathbf{A}_k^{-1} \mathbf{A}_k^{-1\,H} \mathbf{C}_n}{g_k^2}$. We can notice that constrain (3.110) make sure that each user can have only one assigned code.

We consider the optimality in step 4. Every time the algorithm goes to this step, an update of data rate splitting among the multiple codes assigned to the user is needed. Without loss of generosity, we can assume previously, the k th user has been assigned totally $M-1$ codes and the M th code is assigned to it in this iteration. We express the M codes as $\mathbf{C}_1, \mathbf{C}_2, \cdots, \mathbf{C}_M$. Now, our problem is to split the original data rate requirement of the k th user, R_k, into M sub-rate and make each code satisfy one of this sub-rate such that the power of the k th user can be minimized. We can formulate this problem as follows

$$\text{Minimize } \; P_k = \sum_{m=1}^{M} (2^{r_{k,m}} - 1)\lambda_{k,m} \tag{3.113}$$

subject to

$$\sum_{m=1}^{M} r_{k,m} = R_k \tag{3.114}$$

$$r_{k,m} \geq 0 \text{ for all } m \tag{3.115}$$

where $\lambda_{k,m} = \dfrac{\sigma^2 \mathbf{C}_m^H \mathbf{A}_k^{-1} \mathbf{A}_k^{-1H} \mathbf{C}_m}{g_k^2}$.

Since the minimization of original objective is equivalent to the minimization of $\sum_{m=1}^{M} 2^{r_{k,m}} \lambda_{k,m}$, we can transfer our problem to be

$$\text{Minimize } \; f = \sum_{m=1}^{M} \lambda_{k,m} 2^{r_{k,m}} \tag{3.116}$$

subject to the same constraints

$$\sum_{m=1}^{M} r_{k,m} = R_k \tag{3.117}$$

$$r_{k,m} \geq 0 \text{ for all } m \tag{3.118}$$

This is a type of water-filling problem. The optimality can be obtained by the following method.

Since $\sum_{m=1}^{M} r_{k,m} = R_k$, if we do not consider the nonnegative constrain, we can obtain the minimum of objective by

$$\sum_{m=1}^{M} \lambda_{k,m} 2^{r_{k,m}} \geq \sqrt[M]{\prod_{m=1}^{M} \lambda_{k,m} 2^{r_{k,m}}} = \sqrt[M]{(\prod_{m=1}^{M} \lambda_{k,m}) 2^{\sum_{m=1}^{M} r_{k,m}}} = \sqrt[M]{(\prod_{m=1}^{M} \lambda_{k,m}) 2^{R_k}} \tag{3.119}$$

The minimum is achieved when $\lambda_{k,1} 2^{r_{k,1}} = \lambda_{k,2} 2^{r_{k,2}} = \cdots = \lambda_{k,M} 2^{r_{k,M}}$. Thus, we can locate the minimum at

$$r_{k,m} = \frac{R_k + \log_2 \lambda_{k,m}^M - \log_2 \prod_{i=1}^{M} \lambda_{k,m}^i}{M} \tag{3.120}$$

Now, we consider the constraint of nonnegativity. Because of the property of water-filling problem, we can guarantee this constraint and maintain the optimality by simple force the negative items of $r_{k,m}$ to be zero and re-adjust the sub-rate among those positive items of $r_{k,m}$ in the same way as we discussed above. Hence, a recursive algorithm can be developed in the following way

Algorithm 2:

1. Initialization: Setup the variable set as $\mathbf{r}_k = [r_{k,1}, r_{k,2} \cdots, r_{k,M}]$ and the coefficient set as $\boldsymbol{\lambda}_k = [\lambda_{k,1}, \lambda_{k,2}, \cdots \lambda_{k,M}]$. Define the problem dimension as M.
2. According to (3.120), find the M dimension vector $\mathbf{r}_k$ that offers the minimum of the objective function of M variables.
3. Termination check: Count the number of negative elements in $\mathbf{r}_k$ and denote

the number by N. If $N=0$, stop the algorithm and return the variable vector $\mathbf{r}_k$.

4. Update variable set $\mathbf{r}_k$: Pick up the negative elements in $\mathbf{r}_k$ and force them to be zero.
5. Update coefficient set $\boldsymbol{\lambda}_k$: Get rid of the elements in coefficients set λ_m with the same index as the negative items picked up in step 2.
6. Reduce the problem dimension as $M=M-N$ and go to step 2.

Of course, other heuristic approaches to solve the code assignment problem in the non-fully loaded system can also be developed. Since they are all belong the family of sub-optimal approach, due to the limit of the thesis length, we only discuss the above exemplary algorithm. The performance of this approach is given and compared with other approaches in Section 3.6.

3.5 Lower Bound on Optimization

In this section, we find a lower bound on the optimization of the original problem. As shown by (3.28), by Taylor formula, we have convert the original objective function to $\sum_{m=1}^{+\infty}\frac{1}{m!}\sum_{k=1}^{K}\sum_{n=1}^{N}C_{k,n}t_{k,n}^{1-m}x^{m}{}_{k,n}$. The objective is the summation of a set of terms, $\frac{1}{m!}\sum_{k=1}^{K}\sum_{n=1}^{N}C_{k,n}t_{k,n}^{1-m}x^{m}{}_{k,n}$, indexed by m. By exchanging the order of summation and minimization, we can divide the problem into a set of independent minimization problems. Summing all the minima of these sub-problems, a lower bound of the original problem can be obtained because of the following inequality

$$P_{\min}=Min\left\{\sum_{m=1}^{+\infty}\frac{1}{m!}\sum_{k=1}^{K}\sum_{n=1}^{N}C_{k,n}t_{k,n}^{1-m}x^{m}{}_{k,n}\right\}\geq\sum_{m=1}^{+\infty}\frac{1}{m!}Min\left\{\sum_{k=1}^{K}\sum_{n=1}^{N}C_{k,n}t_{k,n}^{1-m}x^{m}{}_{k,n}\right\} \tag{3.121}$$

We consider these sub-problems and try to solve them independently.

- In case of $m=1$

When $m=1$, the problem is very simple of this form

$$\text{Minimize } \sum_{k=1}^{K}\sum_{n=1}^{N} C_{k,n}x_{k,n} \tag{3.122}$$

subject to

$$\sum_{n=1}^{N} x_{k,n} = \hat{R}_k \quad \text{for all } k \tag{3.123}$$

This problem is easy to solve. Evidently, the optimal solution can by obtained by making $x_{k,n} = \hat{R}_k$ where $C_{k,n}$ is the minimum element of the vector $\left[C_{k,1}, C_{k,2}, \cdots C_{k,N}\right]$ for each particular k. The insight behind the solution is that, in order to optimize this sub-problem, each user will pick up the particular best code, i.e., the code with the smallest required power, among the totally N available codes.

- In Case of $m \geq 2$

When $m \geq 2$, by Hölder Inequality [16], we can derive the lower bound for this sub-problem. First, we introduce the Hölder Inequality to readers.

Hölder Inequality

Given $a_i > 0$ and $b_i > 0, i = 1, 2, \cdots, n$, if $k > 0$ and $k \neq 1$, $\frac{1}{k'} + \frac{1}{k} = 1$, then

$$\sum_{i=1}^{n} a_i b_i \leq \left(\sum_{i=1}^{n} a_i^k\right)^{\frac{1}{k}} \left(\sum_{i=1}^{n} b_i^{k'}\right)^{\frac{1}{k'}} \quad \text{for} \quad k > 1 \tag{3.124}$$

$$\sum_{i=1}^{n} a_i b_i \geq \left(\sum_{i=1}^{n} a_i^k\right)^{\frac{1}{k}} \left(\sum_{i=1}^{n} b_i^{k'}\right)^{\frac{1}{k'}} \quad \text{for} \quad k < 1 \tag{3.125}$$

equality holds when $\frac{a_1^{\,k-1}}{b_1} = \frac{a_2^{\,k-1}}{b_2} = \cdots = \frac{a_n^{\,k-1}}{b_n}$

Let $k = \frac{1}{m}$, $k' = \frac{1}{1-m}$, by Hölder Inequality, we have

$$\begin{aligned}\sum_{k=1}^{K}\sum_{n=1}^{N}C_{k,n}t_{k,n}^{1-m}x_{k,n}{}^{m} &\geq \sum_{k=1}^{K}\left\{(\sum_{n=1}^{N}(x_{k,n}^{m})^{\frac{1}{m}})^{m}(\sum_{n=1}^{N}(C_{k,n}t_{k,n}{}^{1-m})^{\frac{1}{1-m}})^{1-m}\right\} \\ &= \sum_{k=1}^{K}\left\{(\sum_{n=1}^{N}x_{k,n})^{m}(\sum_{n=1}^{N}C_{k,n}{}^{\frac{1}{1-m}}t_{k,n})^{1-m}\right\} = \sum_{k=1}^{K}\hat{R}_{k}^{m}(\sum_{n=1}^{N}C_{k,n}{}^{\frac{1}{1-m}}t_{k,n})^{1-m}\end{aligned} \tag{3.126}$$

If we define $T_k \triangleq \sum_{n=1}^{N}C_{k,n}{}^{\frac{1}{1-m}}t_{k,n}$ and apply Hölder Inequality again, we get

$$\begin{aligned}\sum_{k=1}^{K}\hat{R}_{k}^{m}T_{k}^{1-m} &> (\sum_{k=1}^{K}(\hat{R}_{k}^{m})^{\frac{1}{m}})^{m}(\sum_{k=1}^{K}(T_{k}^{1-m})^{\frac{1}{1-m}})^{1-m} = (\sum_{k=1}^{K}\hat{R}_{k})^{m}(\sum_{k=1}^{K}T_{k})^{1-m} \\ &= (\sum_{k=1}^{K}\hat{R}_{k})^{m}(\sum_{k=1}^{K}\sum_{n=1}^{N}C_{k,n}{}^{\frac{1}{1-m}}t_{k,n})^{1-m}\end{aligned} \tag{3.127}$$

According to (3.126-3.127), a very simple method to obtain the minimum of the sub-problem in case of $m \geq 2$ can be developed readily. As we know, $(\sum_{k=1}^{K}\hat{R}_{k})^{m}$ is a constant. Then we only need to focus on the term of $(\sum_{k=1}^{K}\sum_{n=1}^{N}C_{k,n}{}^{\frac{1}{1-m}}t_{k,n})^{1-m}$. As $m \geq 2$ and $1-m \leq 1$, to find the minimum is equivalent to find the maximum of $\sum_{k=1}^{K}\sum_{n=1}^{N}C_{k,n}{}^{\frac{1}{1-m}}t_{k,n}$, given $\sum_{k=1}^{K}t_{k,n} \leq 1$ and $t_{k,n} \geq 0$. Evidently, we can get the maximum by choosing $t_{k,n}$ =1, when $C_{k,n}{}^{\frac{1}{1-m}}$ is the maximum element of the vector $\left[C_{1,n}{}^{\frac{1}{1-m}}, C_{2,n}{}^{\frac{1}{1-m}}\ldots, C_{K,n}{}^{\frac{1}{1-m}}\right]^{T}$ for each n.

Generally speaking, by Taylor expansion, we can divide our original problem into independent sub-problems. The minimum of each sub-problem can be obtained easily according to the above method. Hence, the lower bound of our original problem (3.11-3.15) can be obtained readily by summing the minimum of all the sub-problems.

3.6 Performance Evaluation

3.6.1 Fully Loaded System

In this section, we consider the multicarrier system with the same number of users and code channels. As we discussed in previous section, we can have a near-optimal and a one-to-one assignment approach to make N users time share totally N code channels. In our simulation, we assume all users have the same data rate requirement R. Furthermore, we normalize the data rate by dividing it with the constant B, i.e., we define normalized data rate as $\frac{R}{B}$. As we know, $\frac{R}{B} = \log_2(1+\gamma)$ where γ is the target SNR of each user. Consider too large value of $\frac{R}{B}$ does not make much sense because high SNR target is hard to achieve in wireless communications. We consider the normalize data rate ranges in the interval of $[1,5]$, i.e., the SNR is in the range of $[1,31]$, which is appropriate in wireless mobile environment.

First, we consider a system of 4 users and 4 codes and evaluate the performance of near optimal approach with the interior point method and the one-to-one assignment approach. We plot the average required power against the normalized data rate requirement in Figure 3.4. As shown by this figure, major improvement, about 6 dB power saving, can be obtained by the near optimal approach comparing with the arbitrary time division, which is to make users arbitrarily time share the code channels. We also evaluate the one-to-one assignment approach and notice that performance of the simplified ad hoc approach doesn't degrade much comparing with the near optimal approach. Furthermore, we obtain the curve of the lower bound on the system optimization based on the method discussed in Section 3.5. We can observe that the ad hoc and the near optimal approaches both have performance close to the lower bound with a gap of about 2dB.

We then consider the fully loaded systems with larger system capacity of 8 users and 16 users respectively. We also evaluate the near optimal approach and the one-to-one assignment approach and compare their performance with arbitrary division and the lower bound. As shown by Figure 3.5 and 3.6, we can also conclude that major power saving can be obtained both by near optimal approach and one-to-one assignment. Furthermore, we can observe that the performance of one-to-one assignment becomes closer to the near optimal approach as the system capacity becomes larger. This implies that we can consider the implementation of the simple one-to-one approach in reality instead of the complicated near optimal approach when the system capacity is relatively large.

Another tread of convergence can be observed if we compare Figure 3.4, Figure 3.5, and Figure 3.6 carefully. If we focus on the same data rate requirement, e.g., we choose $R / B = 1$, in Figure 3.3, we can see the difference of the required power with one-to-one approach between the lower bound is about 2dB, while the gap is 1.0dB and 0.7dB respectively when the system capacity is 8 and 16. This implies that that both the near optimal approach and the one-to-one assignment converge to the lower bound when there are more and more users in the system.
We consider the convergence of our near optimal approach and one-to-one approach. Due to the huge computational complexity of the near optimal approach when the system capacity is large, we only evaluate the converge property of the one-to-one assignment. In Figure 3.7, we plot required power difference between the one-to-one approach and the lower bound against different system capacity when the data rate target R / B is 3. We can see that average required power per user with the one-to-one assignment converges to the lower bound, when the number of users in the system increases. Because the required power with one-to-one approach is the upper bound of the near optimal approach, we conclude

the performance of the near optimal approach also converges to lower bound with larger system capacity. Generally speaking, when the system capacity increases, the near optimal approach and one-to-one approach both converge to the lower bound of system optimization.

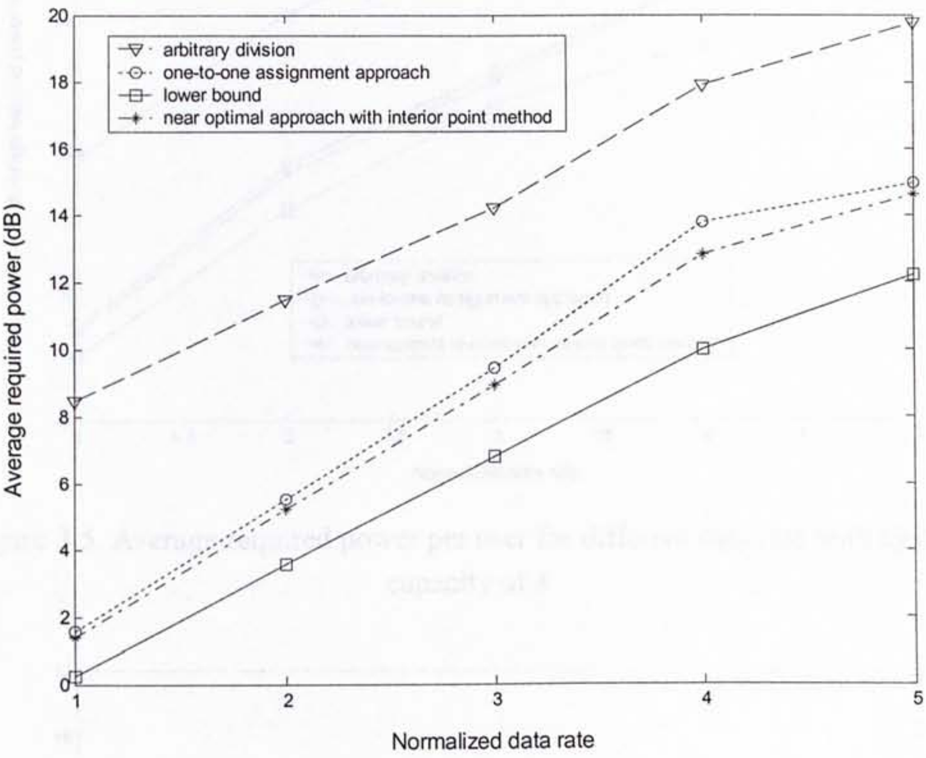

Figure 3.4. Average required power per user for different data rate with system capacity of 4

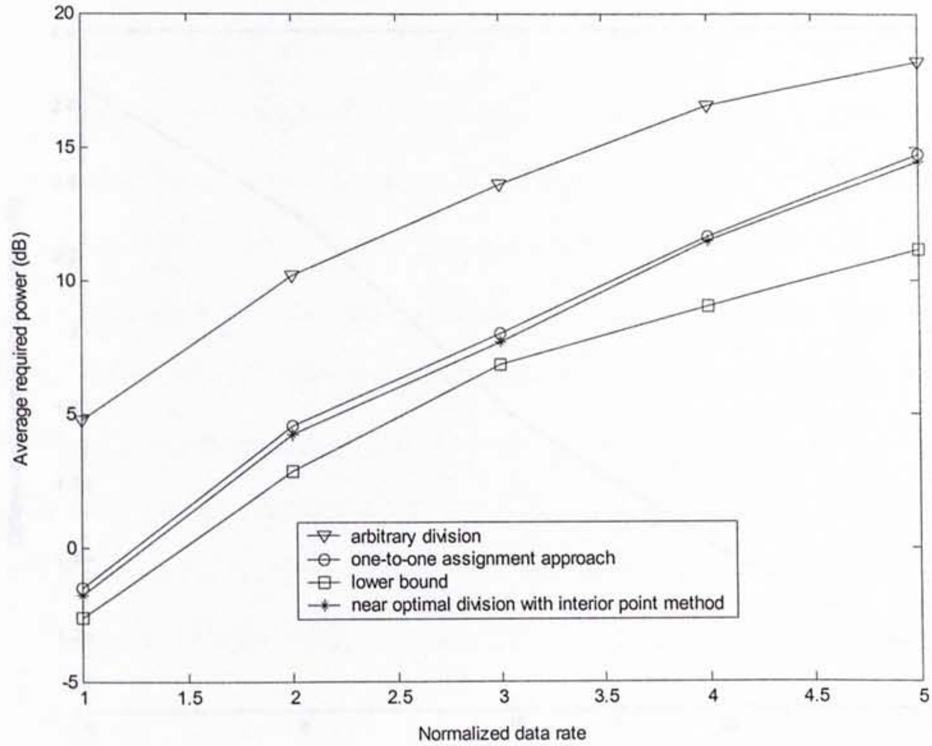

Figure 3.5. Average required power per user for different data rate with system capacity of 8

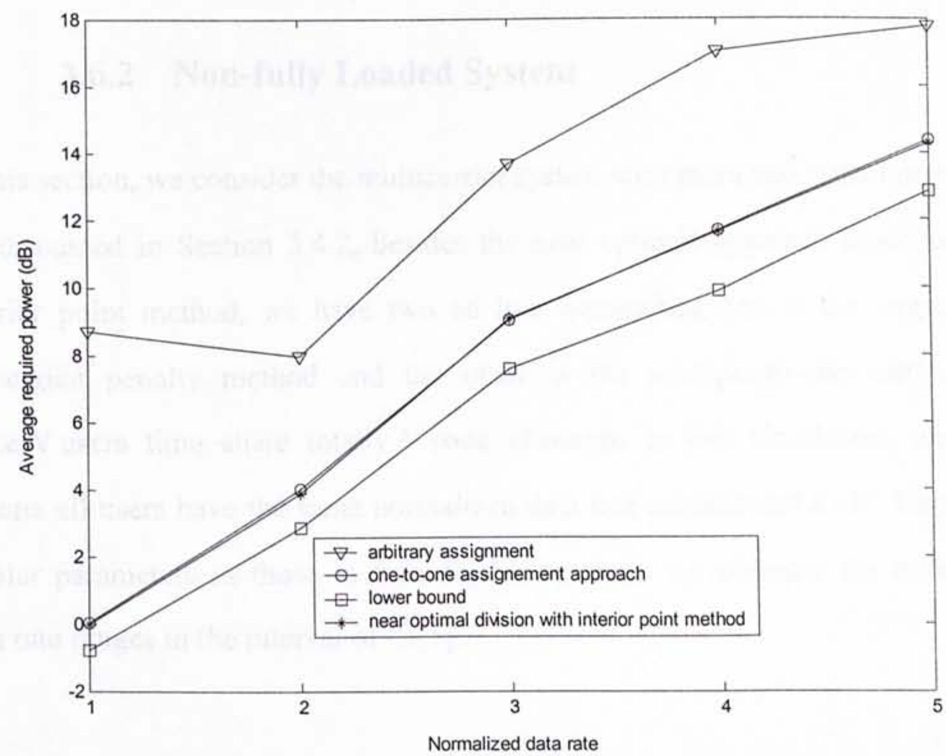

Figure 3.6. Average required power per user for different data rate with system capacity of 16

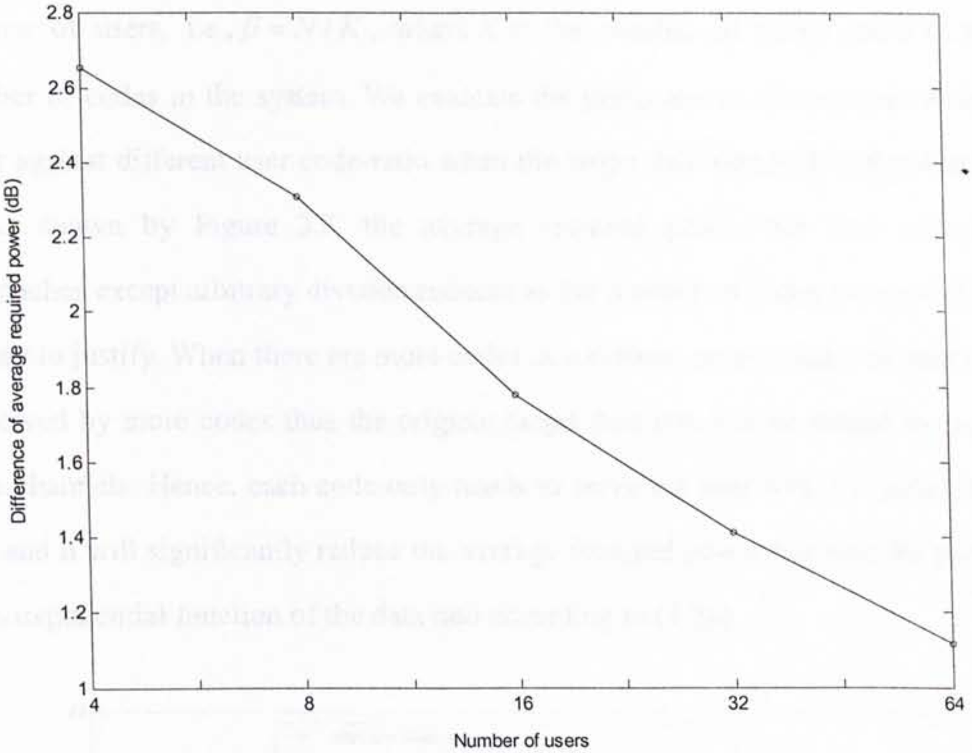

Figure 3.7. Average required power difference between one-to-one approach and lower bound for different system capacity

3.6.2 Non-fully Loaded System

In this section, we consider the multicarrier system with more codes than users. As we discussed in Section 3.4.2, besides the near optimal approach based on the interior point method, we have two ad hoc approaches, one is the augmented lagrangian penalty method and the other is the multiple-to-one method, to make N users time share totally N code channels. In our simulation, we also assume all users have the same normalized data rate requirement R / B. Using the similar parameters as those in fully loaded systems, we consider the normalize data rate ranges in the interval of $[1,5]$.

First, we investigate the effect of the number of codes on the system performance. Intuitively, we guess the performance will improved when there are more codes in the system. We define the user-code-ratio β as the number of codes over the number of users, i.e., $\beta = N/K$, where K is the number of users and N is the number of codes in the system. We evaluate the performance of a system with 4 users against different user-code-ratio when the target data rate R/B is fixed to be 3. As shown by Figure 3.8, the average required power per user with all approaches except arbitrary division reduces as the number of codes increase. This is easy to justify. When there are more codes in a system, on average, one user can be served by more codes thus the original target data rate can be shared by more code channels. Hence, each code only needs to serve the user with a smaller data rate and it will significantly reduce the average required power because the power is an exponential function of the data rate according to (3.98).

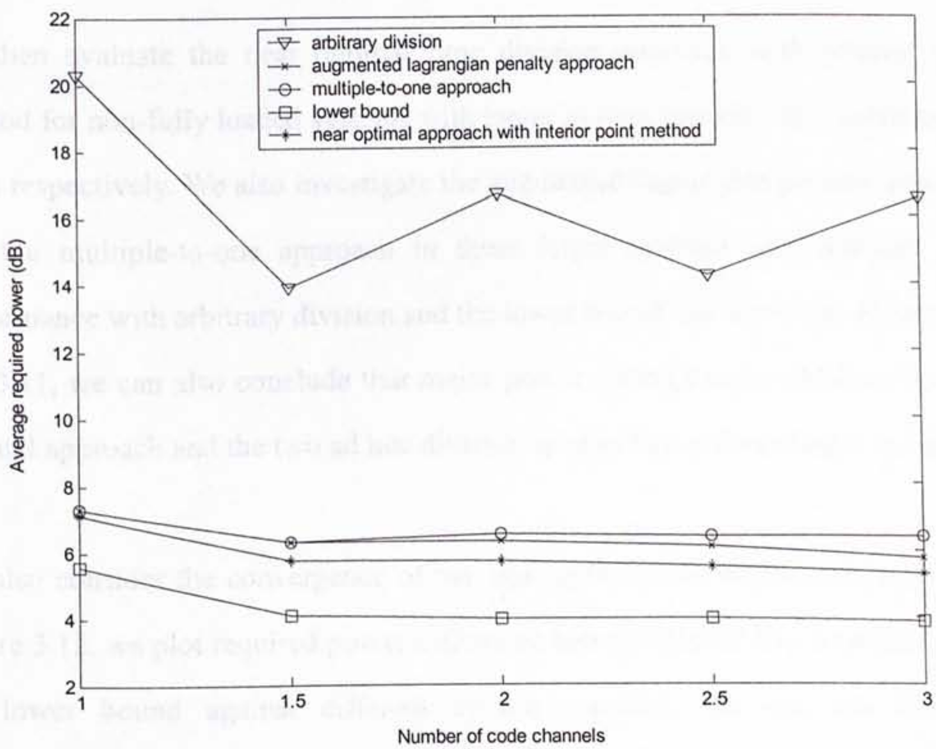

Figure 3.8. Average required power per user against different user-code-ratio β

We then fix the user-code-ratio β to be 1.5 and consider the average required power against the normalized data rate requirement. We evaluate the near optimal approach, augmented lagrangian penalty method, and multiple-to-one method and compare their performance with arbitrary division, which let multiple users share the code channels in time domain arbitrarily. First, we consider a system of 4 users and 6 codes. As shown by this Figure 3.9, with near optimal approach based on the interior point method, we can save 5 to 12 dB power comparing with the arbitrary time division. We also evaluate the augmented lagrangian penalty approach and the multiple-to-one approach. In Figure 3.9, we notice that performance of the two simplified ad hoc approaches are both close to the near optimal approach. Furthermore, we also obtain the lower bound on the system optimization based on the method discussed in Section 3.5. We can observe that the ad hoc and the near optimal approaches all yield performance close to the lower bound with a gap of about 1.6dB for near optimal approach and 1.8 for the ad hoc approaches.

We then evaluate the near optimal time division approach with interior point method for non-fully loaded systems with larger system capacity of 8 users and 16 users respectively. We also investigate the augmented lagrangian penalty approach and the multiple-to-one approach in these larger systems and compare their performance with arbitrary division and the lower bound. As shown by Figure 3.10 and 3.11, we can also conclude that major power saving can be obtained by near optimal approach and the two ad hoc division approaches in these larger systems.

We also consider the convergence of our near optimal and ad hoc approaches. In Figure 3.12, we plot required power difference between the ad hoc approaches and the lower bound against different system capacity. We can see that the performance of the augmented lagrangian penalty method and the multiple-to-one method both converge to the lower bound, when the system capacity becomes larger. As near optimal approach always performs better than ad hoc approaches, we conclude ad hoc approaches and near optimal approach all converge to the lower bound as the system capacity tends to infinity.

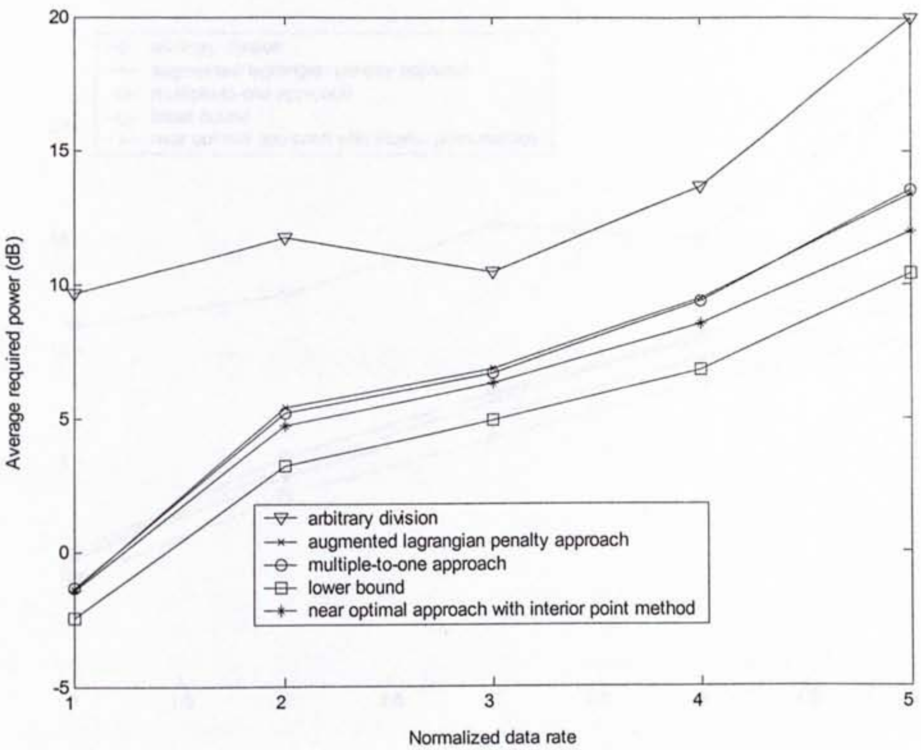

Figure 3.9. Average required power per user for different normalized data rate with system capacity of 4

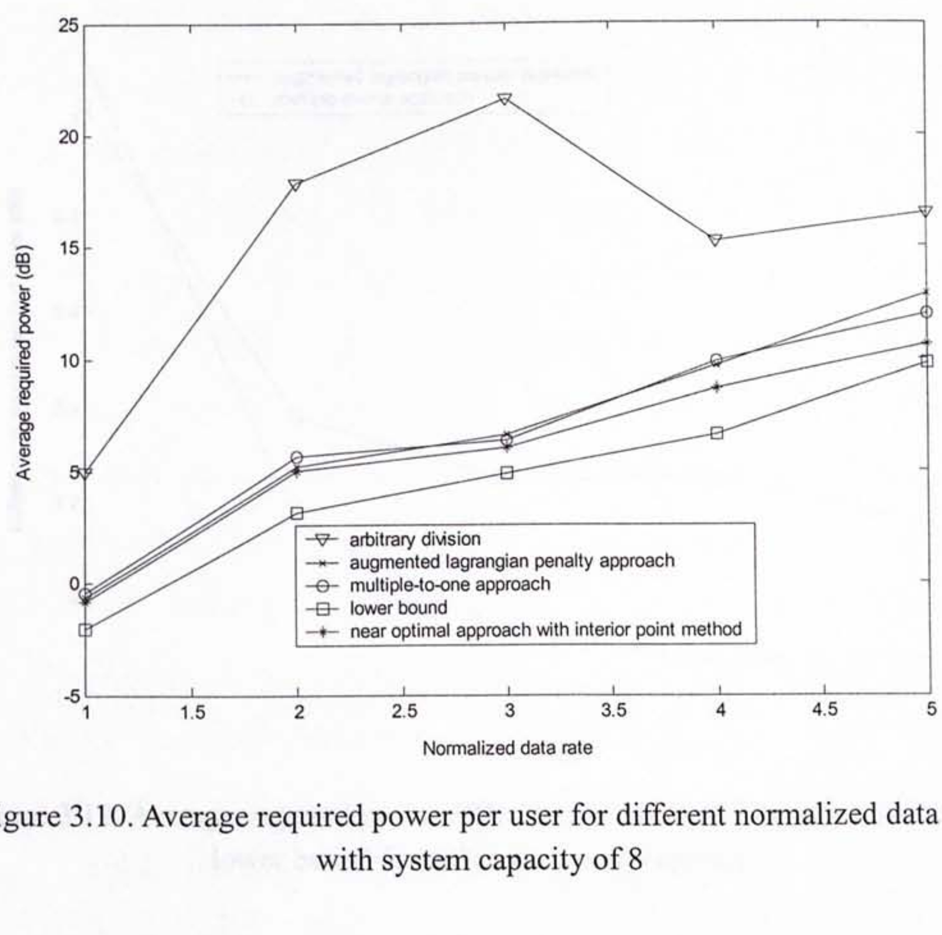

Figure 3.10. Average required power per user for different normalized data rate with system capacity of 8

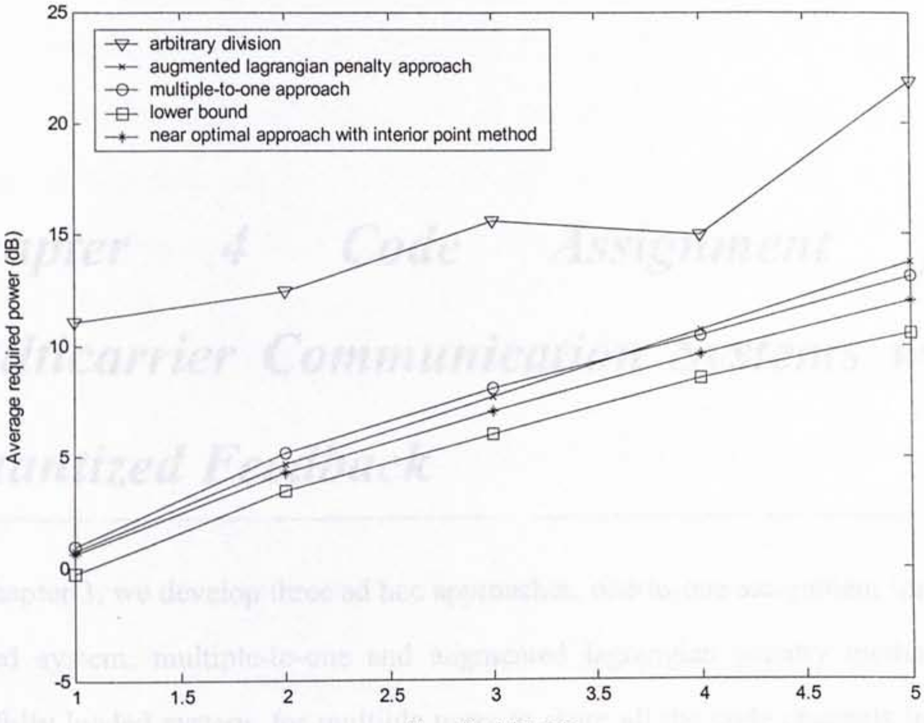

Figure 3.11. Average required power per user for different normalized data rate with system capacity of 16

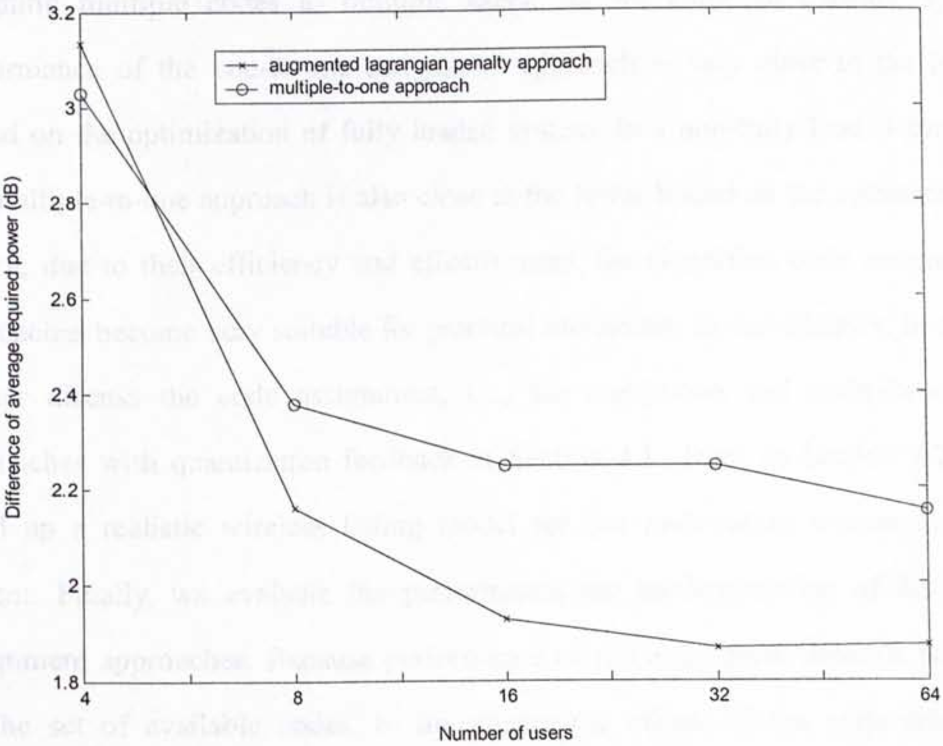

Figure 3.12. Average required power difference between ad hoc approaches and lower bound for different system capacity

Chapter 4 Code Assignment for Multicarrier Communication Systems with Quantized Feedback

In Chapter 3, we develop three ad hoc approaches, one-to-one assignment for fully loaded system, multiple-to-one and augmented lagrangian penalty method for non-fully loaded system, for multiple users to share all the code channels in time domain. We take the one-to-one and multiple-to-one approaches in a family of approach called ***Code Assignment*** Approach because these two methods consider assigning multiple codes to multiple users. As we know in Chapter 3, the performance of the one-to-one assignment approach is very close to the lower bound on the optimization of fully loaded system. In a non-fully loaded system, the multiple-to-one approach is also close to the lower bound on the optimization. Hence, due to their efficiency and effectiveness, the simplified code assignment approaches become very suitable for practical utilization. In this Chapter, first, we further discuss the code assignment, i.e., the one-to-one and multiple-to-one approaches with quantization feedback in Section 4.1. Then, in Section 4.2, we build up a realistic wireless fading model for our multicarrier communication system. Finally, we evaluate the performance the implementation of the code assignment approaches. Because performance of the assignment depends heavily on the set of available codes, to investigate the effects of the code sets, we consider the Hadamard codes as well as random orthogonal codes of different order of selection diversity. Simulations show that random code sets enjoy much better performance with suitable code assignment. For practical implementation, we also have to consider the effects of quantization and the rate of optimization. Results show that only one or two bits are needed for each channel coefficient. The rate of optimization may also be chosen at relatively low frequencies for slow and fast fading respectively, comparing with power control rate.

4.1 Code Assignment for Fully Loaded Systems

As we mentioned in Section 3.4, we can formulate TCDMA problem of a fully loaded multicarrier communication system to an assignment problem, which is a special case of linear programming problem, in the form of

$$\text{Minimize} \quad P=\sum_{k=1}^{K}\sum_{n=1}^{N}t_{k,n}p_{k,n} \tag{4.1}$$

subject to

$$\sum_{n=1}^{N}t_{k,n}=1 \quad \text{for all } k \tag{4.2}$$

$$\sum_{k=1}^{K}t_{k,n}\le 1 \quad \text{for all } n \tag{4.3}$$

$$t_{k,n}\ge 0 \quad \text{for all } k \text{ and } n \tag{4.4}$$

where $p_{k,n}=\dfrac{(2^{R_k/B}-1)\sigma^2\mathbf{C}_n^H\mathbf{A}_k^{-1}\mathbf{A}_k^{-1H}\mathbf{C}_n}{g_k^2}$. As we know, with suitable error control coding, the throughput R_k can reach

$$R_k=B\log_2(1+\gamma_k) \tag{4.5}$$

Thus, we can express $p_{k,n}$ as a function of the SNR requirement of the kth user in form of

$$p_{k,n}=\frac{\gamma_k\sigma^2\mathbf{C}_n^H\mathbf{A}_k^{-1}\mathbf{A}_k^{-1H}\mathbf{C}_n}{g_k^2} \tag{4.6}$$

As we mentioned in Section 3.4.1, this problem is actually a standard form of assignment problem. Hence, the optimal solution can be achieved via linear programming technique or Hungarian method. Therefore, we know the one-one-one assignment is actually the ***optimal code assignment*** scheme for the fully loaded system.

In previous chapter, we assume the wireless channels are Rayleigh fading channels and the fading coefficients, $\alpha_{k,n}$ and g_k^2 for all k and n, vary slowly that so that their values can be accurately estimated. However, in the practical mobile communication system, the fading characteristics are more complicated than our assumptions. Due to user's mobility and the multipath effects, when modeling the small scale fading factor $\alpha_{k,n}$, we need to take into account the Doppler effects and multipath effects at the same time. We also need to consider an appropriate model for the large scale fading coefficient g_k. Then, we consider the estimation and feedback of the fading coefficients. We consider the downlink of the multicarrier communication systems. In this case, the base station will assign the codes to users dynamically in each transmission time slot. However, the channel estimation is performed by every user respectively. Hence, the estimated values of the fading coefficients should be fed back to the base station so that the base station can compute the cost matrix of the required power and determine the code assignment. The assignment scheme can be further explained by Figure 4.1.

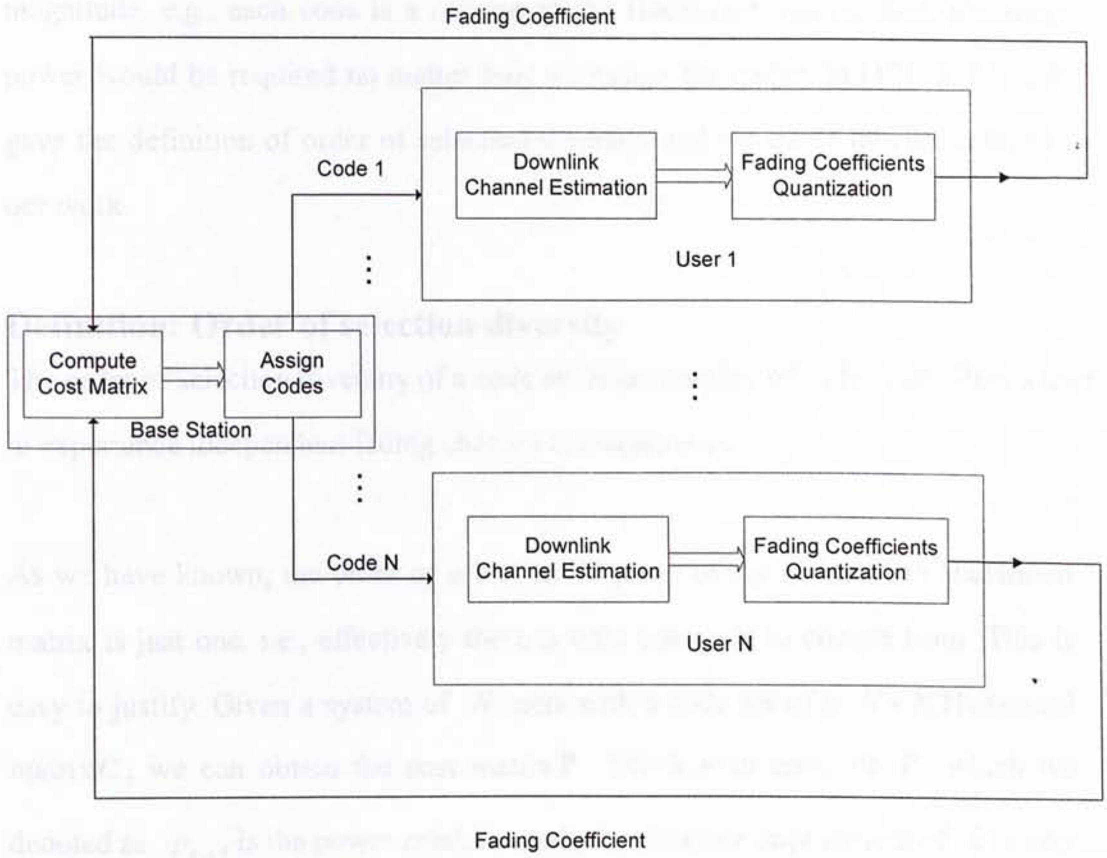

Figure 4.1. Block diagram of code assignment scheme with quantized feedback in fully loaded systems

As shown by Figure 4.1, in the multicarrier communication system, in each time slot, each user will estimate the fading coefficients of the downlink channels. Then, each user will quantize the estimated values and feed it back to the base station via the uplink channel. After base station receives the feedback fading information from all the users, it will compute the cost matrix and obtain the one-to-one code assignment by the linear programming technique or Hungarian method. Then, each code will be assigned corresponding user via the control channel and the data will be transmitted through the data channel of the downlink.

4.2 Random Orthogonal Codes

Notice that if the elements of each code $\mathbf{C}_m$ for $m = 1, 2, \cdots, N$, are of the same magnitude, e.g., each code is a column of the Hadamard matrix, then, the same power would be required no matter how we assign the codes. In [17], T. M. Lok gave the definition of order of selection diversity and we quote this definition in our work.

Definition: Order of selection diversity

The order of selection diversity of a code set is the number of codes that allow a user to experience independent fading channel characteristics.

As we have known, the order of selection diversity of the codes form Hadamard matrix is just one, i.e., effectively there is only one code to choose from. This is easy to justify. Given a system of N users with a code set of a $N \times N$ Hadamard matrix $\mathbf{C}$, we can obtain the cost matrix $\mathbf{P}$. The (k,n) th entry of $\mathbf{P}$, which we denoted as $p_{k,n}$, is the power needed satisfy the data rate requirement of k th user when the n th code is assigned to it. We express $p_{k,n}$ in form of

$$p_{k,n} = \frac{\gamma_k \sigma^2}{g_k^2} \sum_{j=1}^{N} \frac{C_{n,j}^2}{\alpha_{k,j}^2} \tag{4.7}$$

As the codes are columns of Hadamard matrix, we know that $C_{n,j}^2$ is a constant of 1. Hence, we can simplify (4.5) to

$$p_{k,n} = \frac{\gamma_k \sigma^2}{g_k^2} \sum_{j=1}^{N} \frac{1}{\alpha_{k,j}^2} \tag{4.8}$$

We notice that $p_{k,n}$ is irrelevant with the subscript n, i.e., all elements of the vector $[p_{k,1}, p_{k,2}, \cdots, p_{k,N}]$ are the same. In other words, the power required for the k th user is always the same no matter which code is assigned to it.

However, if the elements of each code can have different magnitudes, different performance can be obtained with different code assignments. In this case, we know that each element of the vector $[p_{k,1}, p_{k,2}, \cdots, p_{k,N}]$ is different from the others. This means the kth user can choose totally N different codes and each choice will have different power requirement. Here, the order of selection diversity is N. Diversity gain can, therefore, be achieved through one-to-one assignment.

We consider random orthogonal codes. A random orthogonal code set of N codes can be generated as follows. Consider a randomly generated real matrix $\mathbf{C}$ of dimension $N \times N$. An orthogonal $N \times N$ matrix $\mathbf{Q}$ can be calculated by standard QR decomposition of the form

$$\mathbf{C} = \mathbf{QR} \tag{4.9}$$

where $\mathbf{R}$ is an upper triangular matrix and $\mathbf{Q}$ is an orthogonal matrix, i.e., one satisfying $\mathbf{Q}^T\mathbf{Q} = \mathbf{I}$, where $\mathbf{I}$ is the identity matrix. The columns of $\mathbf{Q}$ can be used as the codes. Then, in general, the performance would differ with different code assignments, i.e., the order of selection diversity of the $N \times N$ random orthogonal codes is N. In the Section 4.4, we would see that the order of selection diversity of the random orthogonal code has a major effect on the performance of the system.

4.3 Wireless Fading Channel Model

To build up an accurate realistic model for the mobile wireless fading channel has drawn much attention of many researchers since nearly thirty years ago. In [18], based on the following equation of the received electrical and magnetic field in wireless fading environment

$$E(w,t) = E_0 \sum_{n=1}^{N} \sum_{m=1}^{M} C_{nm} \cos(wt + w_n t - wT_{nm}), \tag{4.10}$$

where w_n accounts for the Doppler effect and T_{nm} is the delay of multipaths. Without considering the time delay, W. C. Jakes provided a model of flat fading as follows

$$\mu(t) = \mu_1(t) + j\mu_2(t) \tag{4.11}$$

$$\mu_i(t) = \sum_{n=1}^{N} c_{i,n} \cos(2\pi f_{i,n} t + \theta_{i,n}), \quad i = 1,2, \tag{4.12}$$

$$c_{i,n} = \begin{cases} \dfrac{2\sigma_0}{\sqrt{N-\frac{1}{2}}} \sin(\dfrac{\pi n}{N-1}), & n = 1,2,\cdots,N-1, i = 1 \\ \dfrac{2\sigma_0}{\sqrt{N-\frac{1}{2}}} \cos(\dfrac{\pi n}{N-1}), & n = 1,2,\cdots,N-1, i = 2 \\ \dfrac{2\sigma_0}{\sqrt{N-\frac{1}{2}}}, & n = N, \quad i = 1,2 \end{cases} \tag{4.13}$$

$$f_{i,n} = \begin{cases} f_{\max} \cos(\dfrac{\pi n}{N-1}), n = 1,\cdots,N-1, i = 1,2, \\ f_{\max} \quad n = N, \quad i = 1,2 \end{cases} \tag{4.14}$$

$$\theta_{i,n} = 0, \quad n = 1,2,\cdots N, i = 1,2 \tag{4.15}$$

While, in [19], M. Pätzold and F. Laue pointed out that the cross-correlation between the inphase and quadrature components of Jakes' model are significantly from zero. So, in [20], P. Dent slightly modify Jakes' model to gives zero cross-correlation between the real and imaginary parts. Then, in [21], M. F. Pop and N. C. Beaulieu pointed out that Jakes model is nonstationary and they also modify the original model to give a wide-sense stationary (WSS) model.

In this section, we consider a similar model as follows. The small scale fading factor α is modeled as time varying and is given by

$$\alpha(t) = \sqrt{u_c^{\,2}(t) + u_s^{\,2}(t)} \tag{4.16}$$

The functions $u_c(t)$ and $u_s(t)$ account for the effects of multipaths and are given by

$$u_c(t)=\sum_{l=1}^{L}\rho_l\chi_l\cos(2\pi f_l t+\theta_l) \text{ and } u_s(t)=\sum_{l=1}^{L}\rho_l\chi_l\sin(2\pi f_l t+\theta_l) \tag{4.17}$$

where θ_l is the phase of the l th path. It is uniformly distributed in $[0,2\pi)$. ρ_l is the shadowing factor of the l th path. It is modeled as a lognormal random variable with zero mean and 8 dB deviation. χ_k is modeled as the amplitude of the l th path. Since each path contains many mini-path, χ_l can by modeled as a random variable with Rayleigh distribution. f_k is the Doppler frequency of the k th path due to mobility of the users. It is given by

$$f_l=(v/\lambda)\cos(\beta_l) \tag{4.18}$$

where β_l is the incoming angle of the l th path. It is random variables uniformly distributed in $[0,2\pi)$. v is the speed of user and λ is the wavelength of carrier.

We use the widely accepted hata model for the large scale fading factor g. Due to distance propagation, g is given by

$$\begin{aligned} g_{(dB)} &= -(69.55+26.16\log_{10} f_{MHz}-13.82\log_{10} h_1 \\ &\quad -\eta(h_2)+(44.9-6.55\log_{10} h_1)\log_{10} d_{km}-K) \end{aligned} \tag{4.19}$$

where $\eta(\mathrm{h}_2)=3.2(\log_{10}11.75\mathrm{h}_2)^2-4.97$ and $K=0$ for large cities. f_{MHZ} is the carrier frequency. h_1 and h_2 are the height of transmitter and receiver antenna, respectively. d_{km} is the distance between the base station and the users.

4.4 Performance Evaluation of One-to-one Assignment

In this section, we investigate the performance of the multicarrier communication systems with one-to-one assignment via simulation. We will first introduce the parameters used in our simulation and determine their appropriate values.

Evidently, quantization will have effect on the performance of the system. Spontaneously, we can expect that performance depends on the number of bits used in quantization. Also, the performance will depend on the optimization rate, i.e., the frequency for the base station to update the code assignment. In this section, we will also investigate the system performance with different codes of different order of selection diversity.

4.4.1 Parameters Setup

In the simulations, we consider the downlink of the cell mobile system. We only consider code assignment with one base station. We assume that users are uniformly located in a 120 degree sector with cell radius of 1 km. The mobile channel fading model is the model discussed in previous section. We also assume the variance of the AWGN contribution is normalized to one unit. The code sets, unless otherwise stated, are the random orthogonal codes generated from QR decomposition. In our simulations, we investigate two speeds of user mobility, more specifically, 3.6 km/h and 72 km/h, which are typical pedestrian speed and driving speed corresponding to slow and fast fading, respectively. Other parameters used in the simulations are listed as follows

- Number of multipaths L : 20
- Transmitter antenna height h_1 : 30 m
- Receiver antenna height h_2 : 1.5 m
- RF frequency f_{MHz} : 900 MHz
- Cell radius : 1 km
- Power control rate : 1000 Hz
- Number of users in system : 4
- Number of code channels : 4

4.4.2 Effect of Quantization

In practice, in order to perform the optimization, the channel coefficients must be quantized and fed back to the base station. We first consider the performance with different numbers of bits per coefficient used in the quantization. In simulations, we investigate the performance of PCM quantization, DPCM quantization, and the perfect feedback, which assumes the exact value of the fading coefficient can be perfectly feed back to the base station. Figure 4.2 shows the simulation results with four code channels and four users. The target SNR is 10 dB for each user and the speed of each user is 3.6 km/h, which means the fading is slow fading. As shown in the figure, with the one-to-one code assignment, we can save at least 6 dB of power to achieve the target SNR, comparing with the system with arbitrary code assignment. In Figure 4.2, we also show the system performance with Hadamard codes for comparison. We can see that the performance with Hadamard code is close with arbitrary assignment because the order of selection diversity of Hadamard code is one and the assignment can not improve the performance in this case.

If we use more feedback bits to quantize the fading coefficients, the system performance will approach the optimal situation with perfect feedback information at the cost of system complexity. By making a tradeoff between the performance and the complexity, in the case of slow fading, Figure 4.2 shows that one bit quantization per coefficient is a good choice. Furthermore, if we introduce Differential Pulse Code Modulation (DPCM) into the quantization process, the performance will improve.

Next, we consider the performance in fast fading channels with four code channels and four users. The target SNR for each user is also 10 dB and the speed of each user is 72 km/h. The optimization rate is also 1000 Hz. As shown in Figure 4.3, one-to-one code assignment provides much gain comparing with arbitrary assignment or using Hadamard codes. Due to the fast change of the fading, usually, we need two bits in quantization. Figure 4.3 also shows that the performance of DPCM is better than that of PCM.

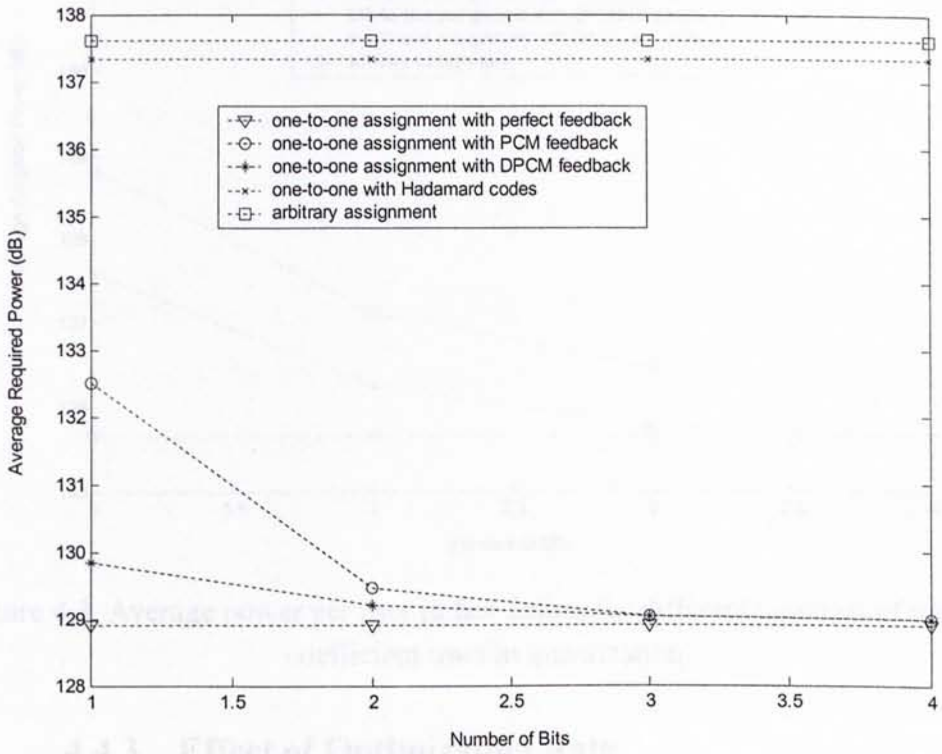

Figure 4.2. Average power per user in slow fading for different numbers of bits per coefficient used in quantization

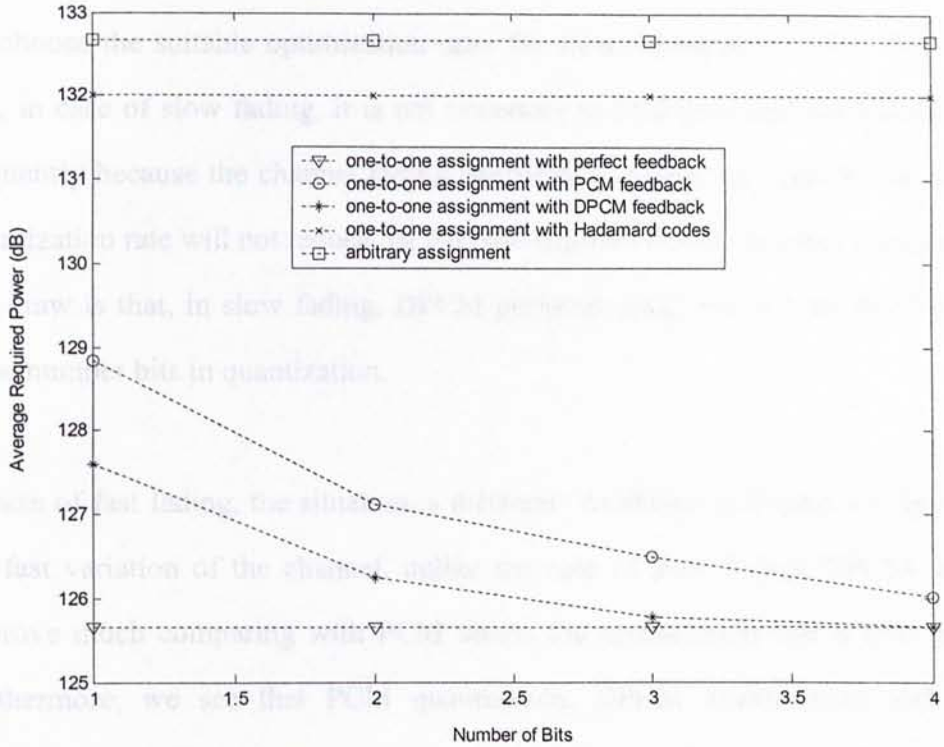

Figure 4.3. Average power per user in fast fading for different numbers of bits per coefficient used in quantization

4.4.3 Effect of Optimization Rate

In our channel model, although the channel is varying over time, in a short time slot, each fading coefficient may approximately remain constant. Therefore, we do not need to feed back the coefficients and update the code assignment as frequently as the power control rate. Evidently, this will cause degrade of system performance. However, lower optimization rate can significantly reduce system complexity. In this section, we investigate performance of PCM quantization, DPCM quantization, and perfect feedback under different optimization rate and try to find the appropriate rate for slow fading and fast fading channels, respectively. As we mentioned in previous section, we use one bit in quantization in slow fading while two bits for fast fading. As shown in Figure 4.4, the performance of the PCM is nearly the same under different optimization rate. The performance with DPCM and perfect feedback were improved when the optimization increases from 31.25 Hz to 62.5 Hz and are both hardly to be improved with further higher optimization rate. Therefore, we choose the suitable optimization rates for slow fading as 62.5 Hz. We can see that, in case of slow fading, it is not necessary to feed back and optimization very frequently because the channel fading coefficients do not vary fast. Using a higher optimization rate will not reduce the average required power. Another conclusion we can draw is that, in slow fading, DPCM performs much better than PCM with the same number bits in quantization.

In case of fast fading, the situation is different. As shown in Figure 4.5, because of the fast variation of the channel, unlike the case of slow fading, DPCM does not improve much comparing with PCM unless the optimization rate is over 500 Hz. Furthermore, we see that PCM quantization, DPCM quantization, and perfect feedback all yield performance poor with lower optimization rate. There is only about 1 dB power saving with optimization rate of 31.25 Hz. We can observe that, in Figure 4.5, the performance improved much as the optimization rate increase from 31.15 Hz to 500 Hz. The required power is reduce about 4 dB with optimization rate of 500 Hz, comparing with arbitrary assignment and assignment with Hadamard codes. While, when we optimize the system with rate of 1000 Hz, as shown by Figure 4.5, there is no very much improvement on the performance. Therefore, by making tradeoff between the performance and system complexity, we choose 500 Hz as the appropriate optimization rate for the fast fading.

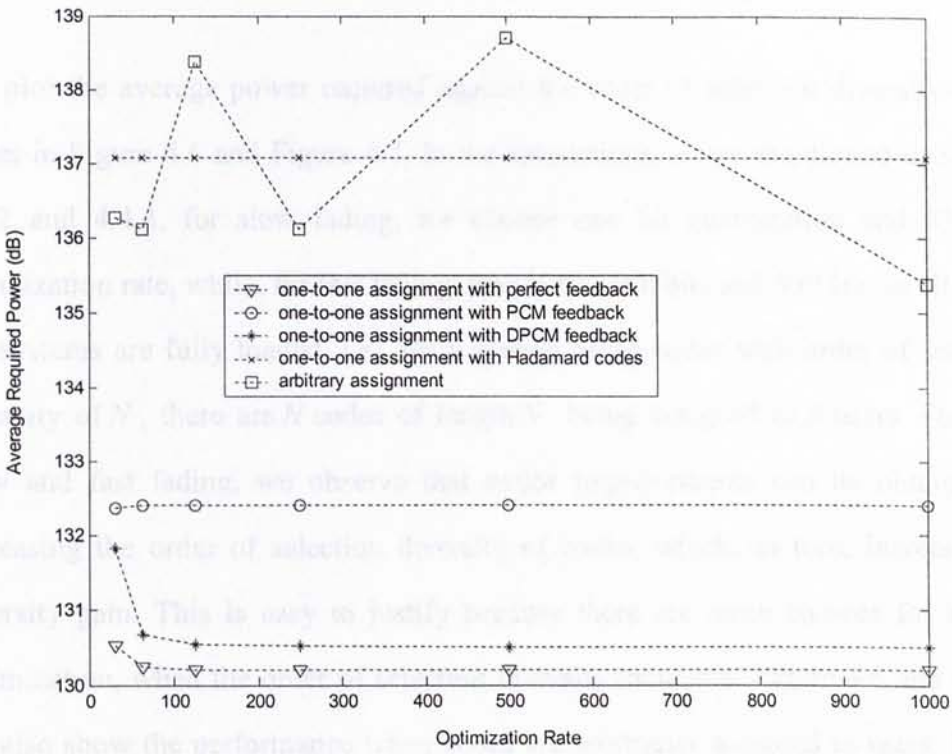

Figure 4.4. Average power per user in slow fading for different feedback and optimization rate

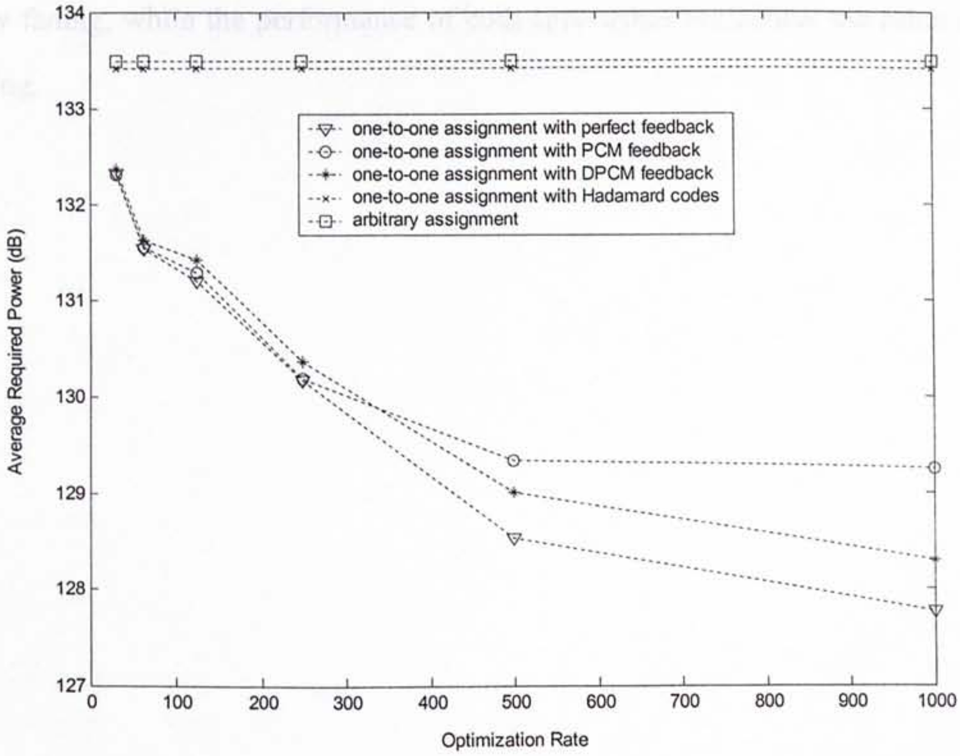

Figure 4.5. Average power per user in fast fading for different feedback and optimization rate

4.4.4 Effect of Order of Selection Diversity

We plot the average power required against the order of selection diversity of the codes in Figure 4.6 and Figure 4.7. In the simulations, as we mentioned in Section 4.4.2 and 4.4.3, for slow fading, we choose one bit quantization and 62.5 Hz optimization rate, while, for fast fading, we choose two bits and 500 Hz. In all cases, the systems are fully loaded, i.e., for a system using codes with order of selection diversity of N, there are N codes of length N being assigned to N users. For both slow and fast fading, we observe that major improvements can be obtained by increasing the order of selection diversity of codes, which, in turn, increases the diversity gain. This is easy to justify because there are more choices for system optimization, when the order of selection diversity increases. Figure 4.6 and Figure 4.7 also show the performance when codes are arbitrarily assigned to users. Notice that the order of selection diversity has little effect if the codes are arbitrarily assigned. We can also observe that DPCM gives better performance than PCM in slow fading, while the performance of both approaches are almost the same in fast fading.

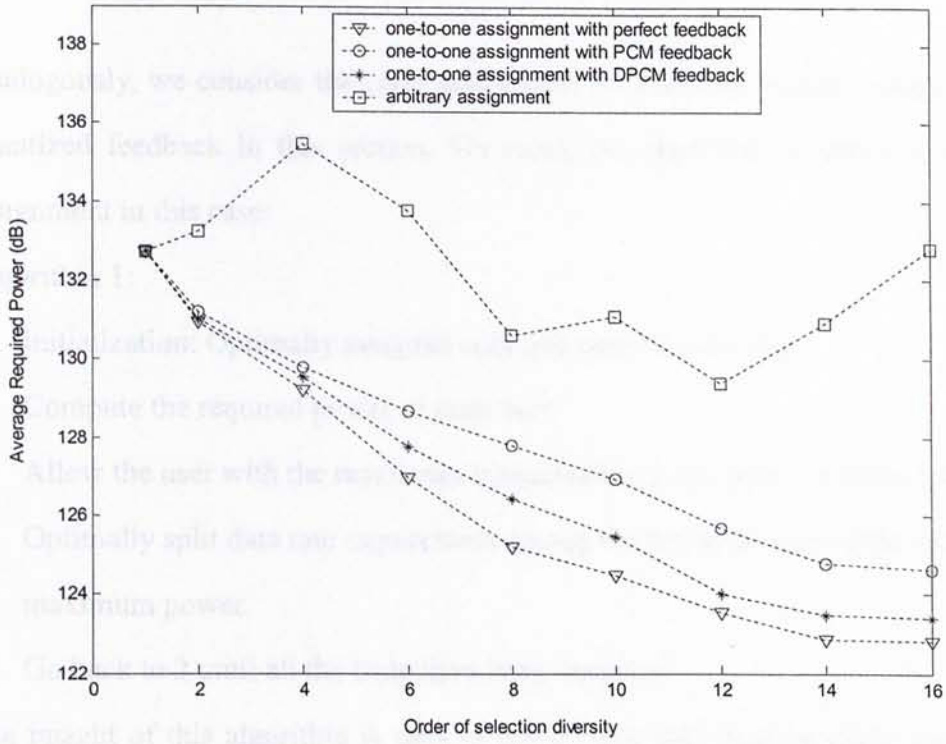

Figure 4.6. Average power per user in slow fading for codes with different order of selection diversity

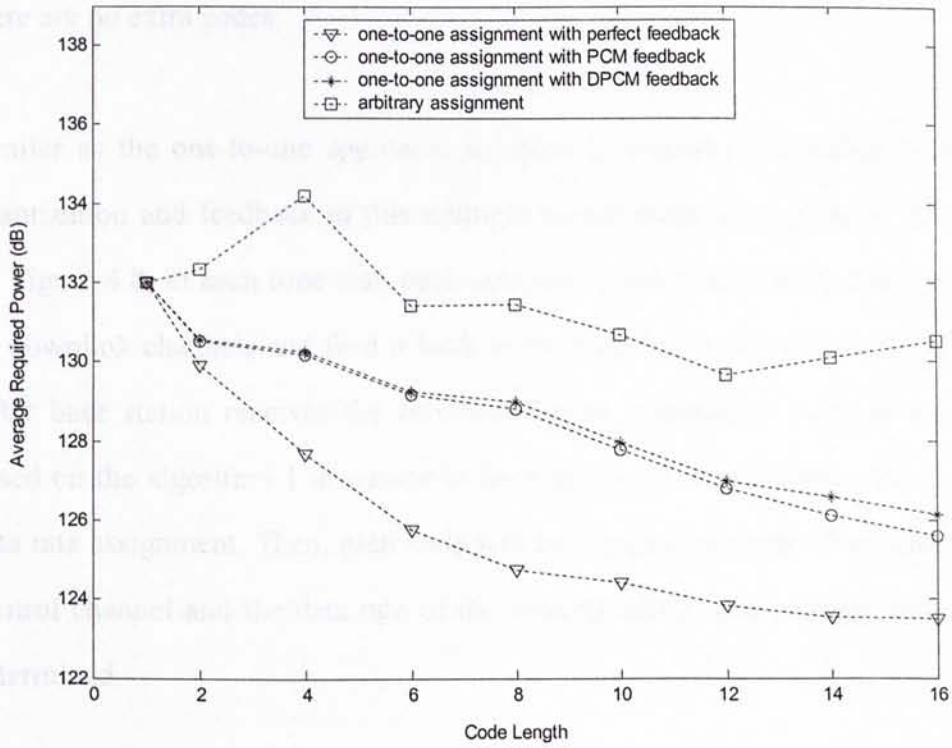

Figure 4.7. Average power per user in fast fading for codes with different order of selection diversity

4.5 Code Assignment for Non-fully Loaded Systems

Analogously, we consider the code assignment for non-fully loaded systems with quantized feedback in this section. We recall the algorithm to obtain the code assignment in this case:

Algorithm 1:

1. Initialization: Optimally assigned only one code to each user.
2. Compute the required power of each user.
3. Allow the user with the maximum transmission power pick one more code.
4. Optimally split data rate requirement among the multiple code of the user with maximum power.
5. Go back to 2 until all the code have been assigned.

The insight of this algorithm is easy to justify. The initialization allow each user optimally choose one code first. Then, the following iterative steps will assign more codes to the user with maximum power requirement to reduce its power until there are no extra codes.

Similar as the one-to-one approach, we need to consider the fading coefficient quantization and feedback in this multiple-to-one assignment scheme. As shown by Figure 4.8, in each time slot, each user will quantize the fading coefficients of its downlink channels and feed it back to the base station via the uplink channel.. After base station receives the feedback fading information from all the users, based on the algorithm 1 discussed in Section 3.4.2.3, it will obtain the code and data rate assignment. Then, each code will be assigned corresponding user via the control channel and the data rate of the corresponding code channel will also be determined.

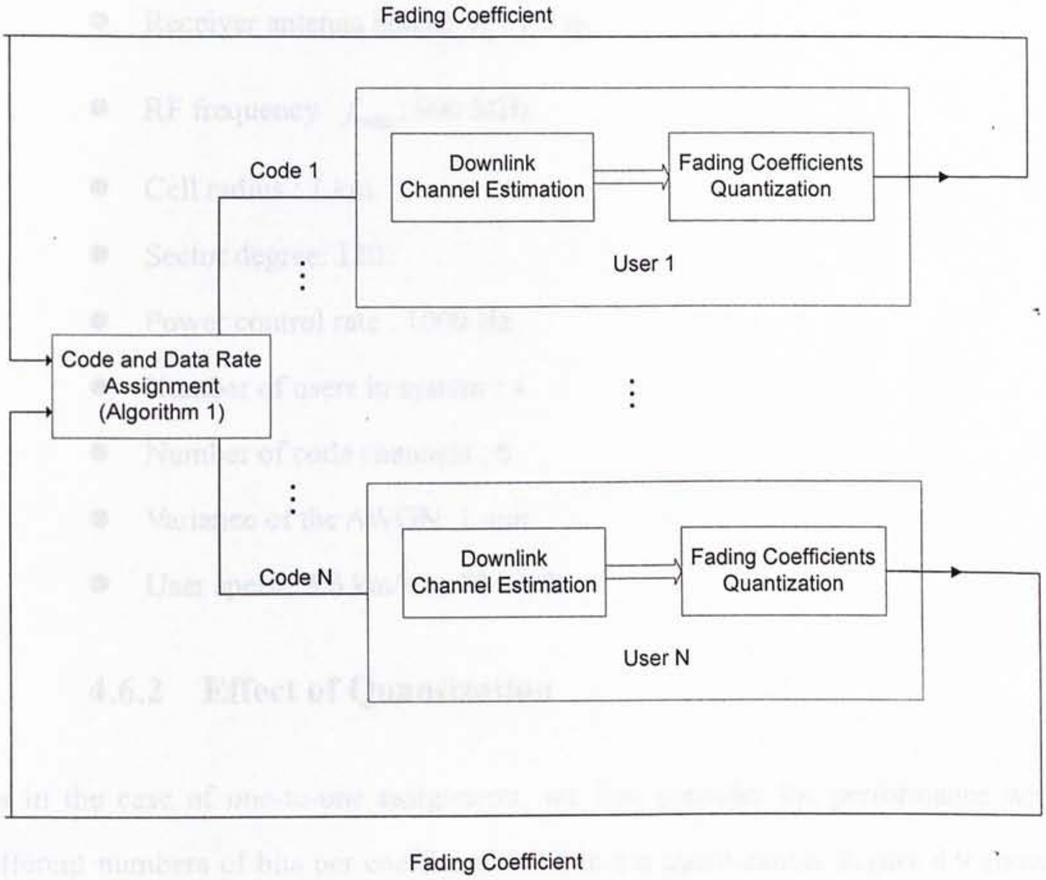

Figure 4.8. Block diagram of code assignment scheme for non-fully loaded systems

4.6 Performance Evaluation of Multiple-to-one Assignment

4.6.1 Parameters Setup

We investigate the performance of multiple-to-one assignment approach in the environment of realistic wireless fading channels. We adopt the fading model in Section 4.3. We use nearly the same parameters as in the case of fully-loaded system and relist them as follows:

- Code set: Random orthogonal codes
- Normalized data rate target:3
- Number of multipaths L : 20
- Transmitter antenna height h_1 : 30 m

- Receiver antenna height h_2 : 1.5 m
- RF frequency f_{MHz} : 900 MHz
- Cell radius : 1 km
- Sector degree: 120
- Power control rate : 1000 Hz
- Number of users in system : 4
- Number of code channels : 6
- Variance of the AWGN: 1 unit
- User speed: 3.6 km/h or 72 km/h

4.6.2 Effect of Quantization

As in the case of one-to-one assignment, we first consider the performance with different numbers of bits per coefficient used in the quantization. Figure 4.9 shows the results of quantization with different number of bits in case of slow fading, i.e., the user's speed is 3.6 km/h. As shown in the figure, with the multiple-to-one assignment, we can save at least 4 dB of power to achieve the target SNR, comparing with the system with arbitrary code assignment.

If we use more feedback bits to quantize the fading coefficients, the system performance will approach the optimal situation with perfect feedback information at the cost of system complexity. By making a tradeoff between the performance and the complexity, in the case of slow fading, Figure 4.9 shows that one bit quantization per coefficient is a good choice. Furthermore, as the case of fully loaded system, if we introduce Differential Pulse Code Modulation (DPCM) into the quantization process, the performance will improve.

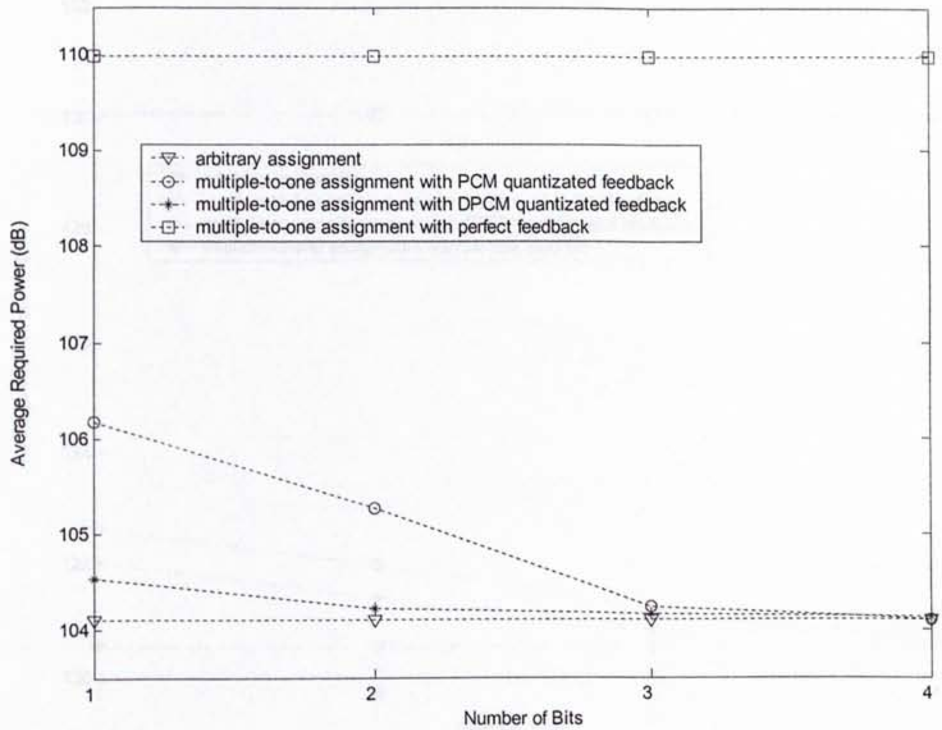

Figure 4.9. Average power per user in fast fading for different numbers of bits per coefficient used in quantization

Next, we consider the performance in fast fading channels with four code channels and four users. The target SNR for each user is also 10 dB and the speed of each user is 72 km/h. The optimization rate is also 1000 Hz. As shown in Figure 4.10, multiple-to-one code assignment provides much gain comparing with arbitrary assignment. Due to the fast change of the fading, usually, we need two bits in quantization. Figure 4.3 also shows that the performance of DPCM is better than that of PCM.

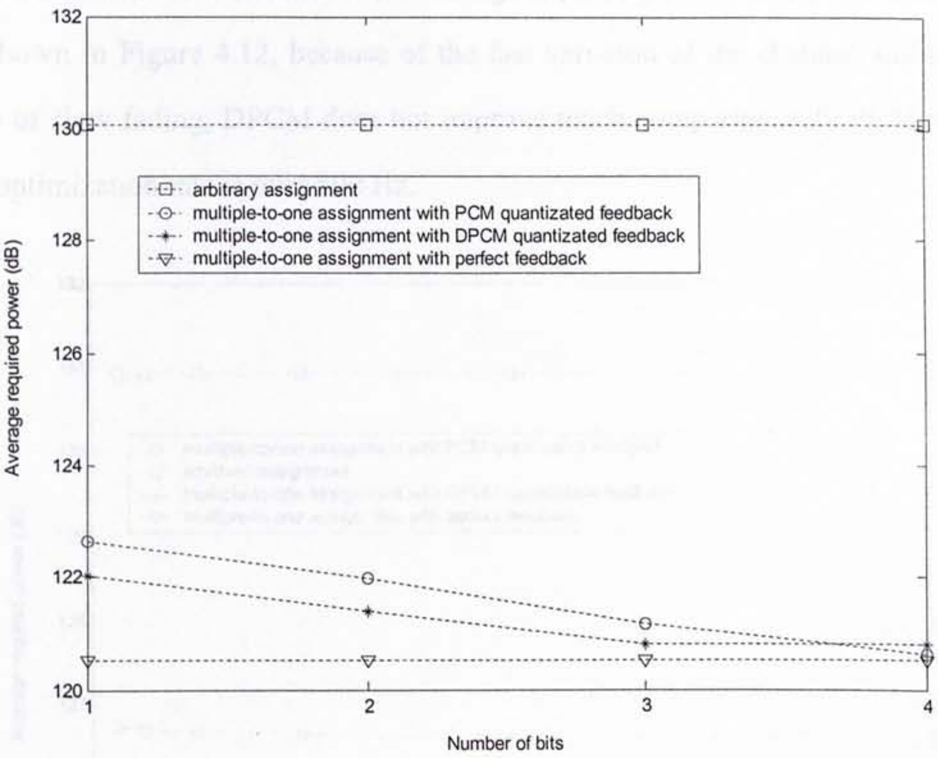

Figure 4.10. Average power per user in fast fading for different numbers of bits per coefficient used in quantization

4.6.3 Effect of Optimization Rate

As we have mentioned in Section 4.4.3, in our channel model, although the channel is varying over time, in a short time slot, each fading coefficient may approximately remain constant. Therefore, we do not need to feed back the coefficients and update the code assignment as frequently as the power control rate. In order to trade off appropriate between the system performance and complexity, in this section, we investigate the effect of optimization rate on the non-fully loaded system performance and try to find the appropriate rate for slow fading and fast fading channels, respectively. As shown in Figure 4.11 and Figure 4.12, when using one bit quantization for slow fading and 2 bits for fast fading respectively, the suitable optimization rates for slow fading and fast fading are about 125 Hz and 500 Hz, respectively. We can see that, in case of slow fading, it is not necessary to feed back and optimization very frequently. Using a higher optimization rate will not reduce the average required power. In case of fast fading, as shown in Figure 4.12, because of the fast variation of the channel, unlike the case of slow fading, DPCM does not improve much comparing with PCM unless the optimization rate is over 500 Hz.

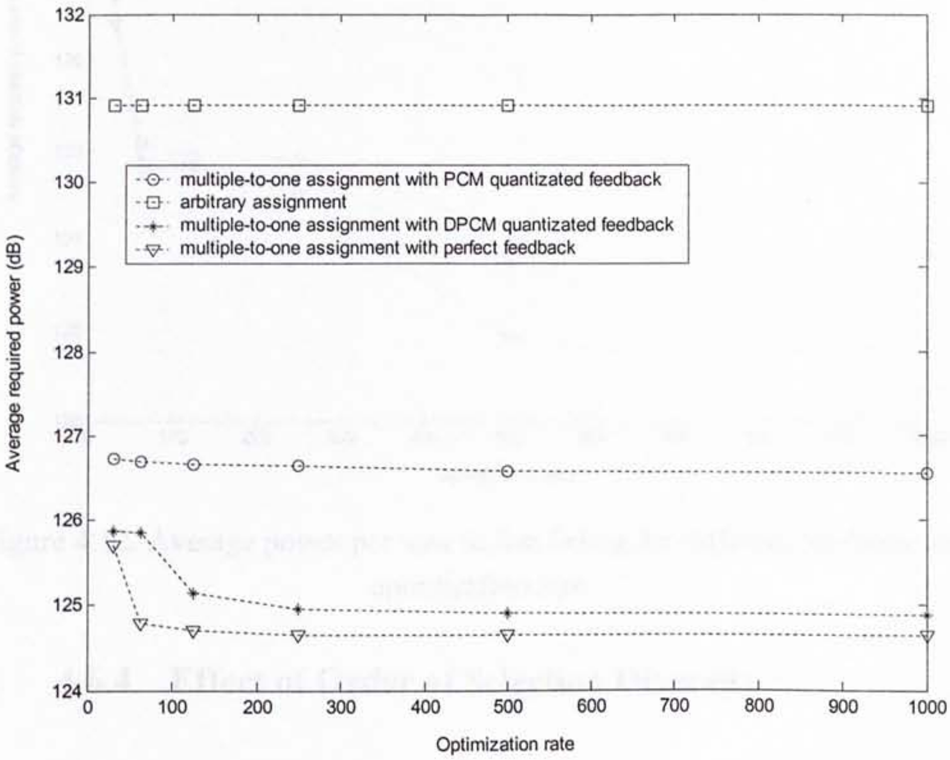

Figure 4.11. Average power per user in slow fading for different feedback and optimization rate

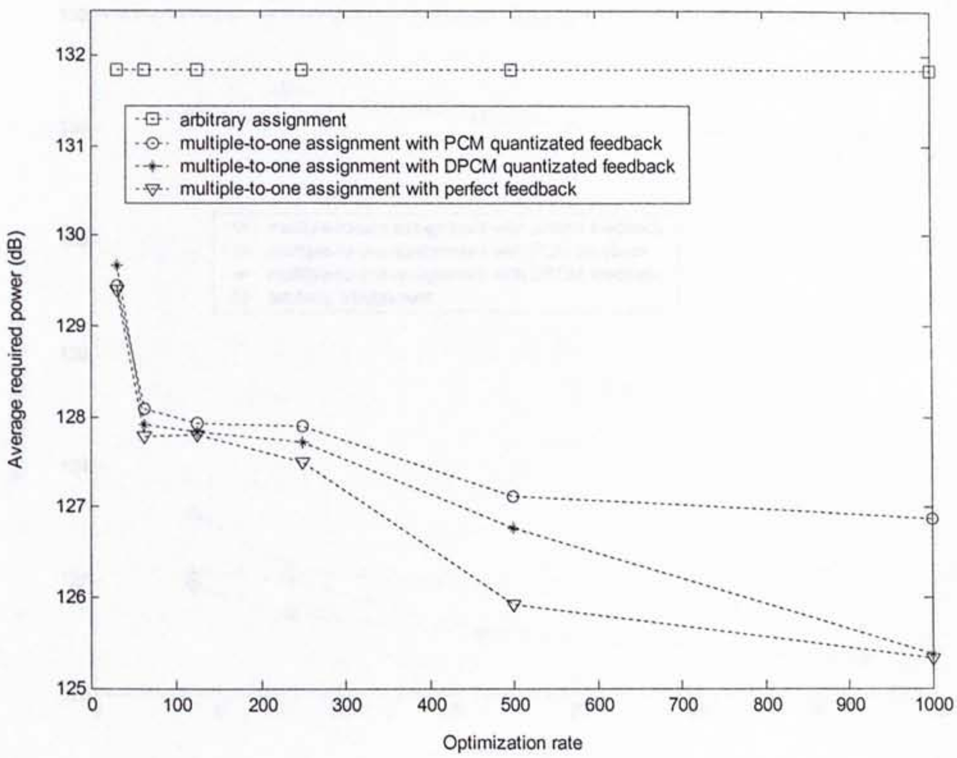

Figure 4.12. Average power per user in fast fading for different feedback and optimization rate

4.6.4 Effect of Order of Selection Diversity

We plot the average power required against the order of selection diversity of the codes in Figure 4.13 and Figure 4.14. In the simulations, for slow fading, we choose one bit quantization and 62.5 Hz optimization rate, while, for fast fading, we choose two bits and 500 Hz. In all simulation, we consider a system of 4 users. For both slow and fast fading, we observe that improvements can be obtained by increasing the order of selection diversity of codes, which, in turn, increases the diversity gain. This is easy to justify because there are more choices for system optimization, when the order of selection diversity increases. Figure 4.13 and Figure 4.14 also show the performance when codes are arbitrarily assigned to users. Notice that the order of selection diversity has little effect if the codes are arbitrarily assigned.

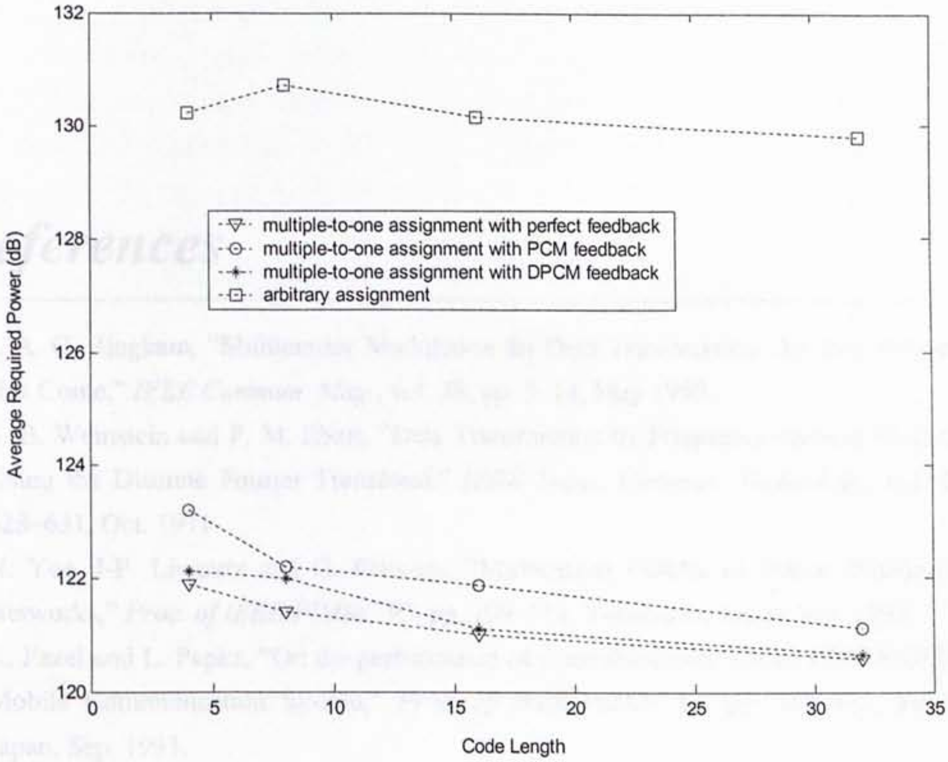

Figure 13. Average power per user in slow fading for codes with different order of selection diversity

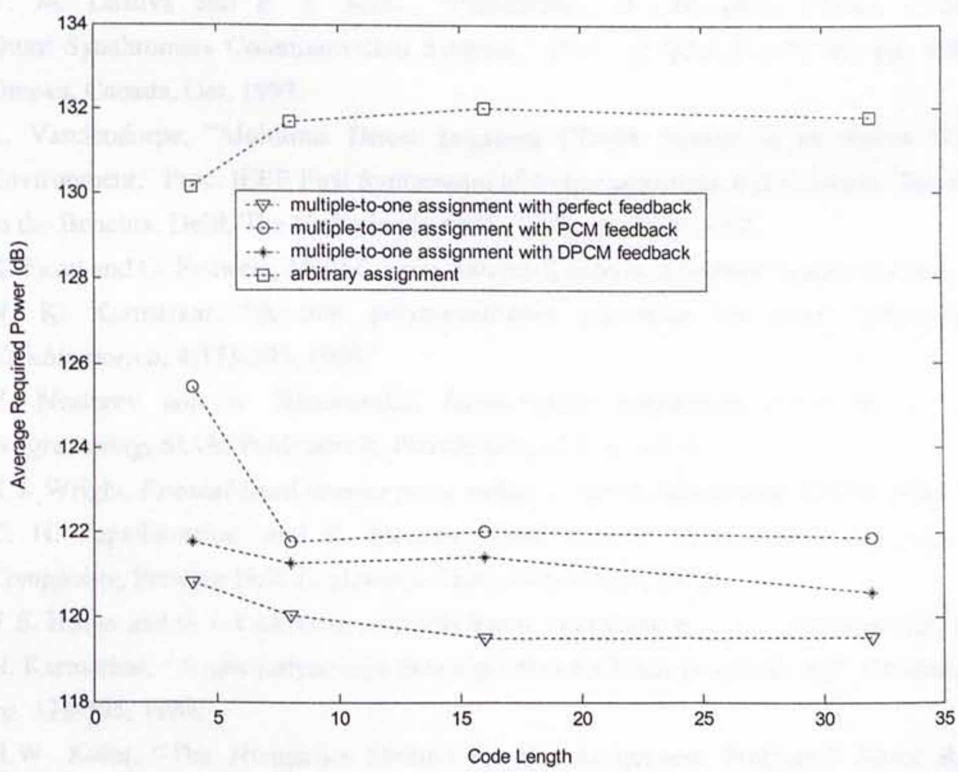

Figure 13. Average power per user in fast fading for codes with different order of selection diversity

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